Direct Trajectory Optimized Walking Gaits Implemented on a Planar Biped Via Virtual Constraints

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1 Introduction

The generation of useful walking gaits is critical to the development of legged robots, but there are many approaches and little consensus as to which is preferable. Gait generation via direct trajectory optimization can leverage sparsity-exploiting NLP solvers to efficiently find dynamically consistent system trajectories [1]. However, this method works best when constraints are analytical and sparse, making it incompatible with the complex stability constraint that underlies the hybrid zero dynamics (HZD) method pioneered by Westervelt [3]. The HZD approach employs virtual constraint control to yield mathematically stable gaits and has been shown to be robust in experiment to some modeling errors [4] and even augmentation with complementary control strategies [5, 6]. However, success in hardware often requires gait shaping constraints to cope with practical factors such as imperfect joint tracking, modeling uncertainty, and state estimation, leading to questions about the usefulness of such stability analysis. In this paper, the success of gaits generated without regard to HZD stability is explored. Gaits are generated via direct trajectory optimization, encoded as a set of virtual constraints, and successfully implemented on the planar biped ERNIE (Fig. 1).

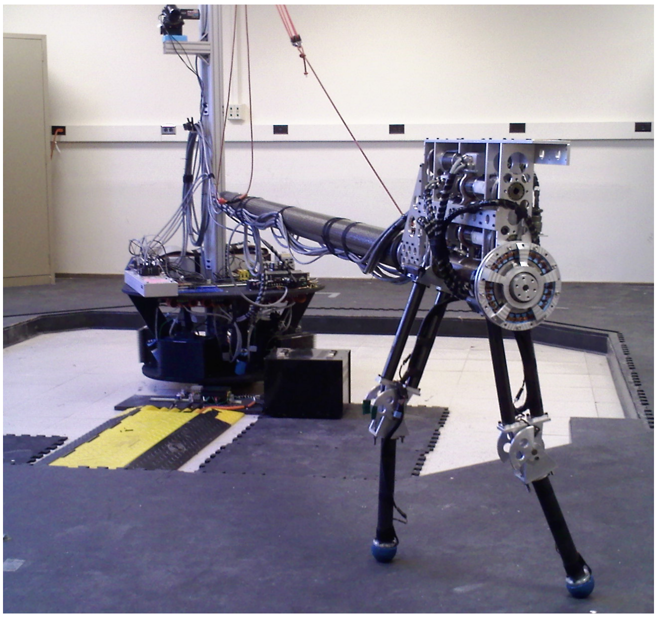

Figure 1: ERNIE, the planar biped used to generate experimental results. Gaits generated via direct trajectory optimization showed experimental reliability similar to HZD-based gaits on this platform.

2 Gait Generation

Direct trajectory optimization is a collocation method that involves the simultaneous optimization of state (q and $\dot{q}$), control (u), and contact force (λ) trajectories. The trajectory is discretized into waypoints separated by time steps of size h, with the equations of motion encoded as optimization constraints.

$$\mathbf{q_i} - \mathbf{q_{i-1}} - h\dot{\mathbf{q}}_\mathbf{i} = 0 \qquad (1)$$

$$M_i[\dot{\mathbf{q}}_\mathbf{i} - \dot{\mathbf{q}}_\mathbf{i-1}] + h[C_i\dot{\mathbf{q}}_\mathbf{i} + G_i(\mathbf{q_i}) - B\mathbf{u_i} + R_i\lambda_\mathbf{i}] = 0, \qquad (2)$$

with mass matrix (M), Coriolis matrix (C), gravity vector (G), control matrix (B), and contact matrix (R). Ground contact is handled by a set of contact constraints

$$\Phi(\mathbf{q_i}) \geq 0 \qquad (3)$$

$$\Phi(\mathbf{q_i}) \cdot \lambda_{i,z} = 0 \qquad (4)$$

$$R_i^T \dot{\mathbf{q}}_\mathbf{i} \cdot \lambda_{i,z} = 0 \qquad (5)$$

$$\lambda_{i,x} - \mu\lambda_{i,z} \leq 0 \qquad (6)$$

$$\lambda_{i,z} > 0 \qquad (7)$$

$$\lambda_{1,z} + \lambda_{2,z} > \lambda_{min}, \qquad (8)$$

which enforce non-penetration (3), force only during contact (4), zero contact velocity during contact (5), Coloumb friction (6), positive contact forces (7), and no flight (8). Gradients are computed analytically, and the resulting sparse gradient matrix enables the problem to be efficiently solved with state-of-the-art gradient-based optimization software. A key advantage of this approach is that it is completely general. It can be applied to essentially any system, which facilitates fair comparisons of gaits generated for different robot designs, and to generate any type of gait (periodic, aperiodic, etc.). For example, the approach has been used to compare gaits of a planar, underactuated biped both with and without its use of a reaction wheel actuation system in the body [2].

3 Experimental Implementation

Gaits were implemented in experiment using the same method of virtual constraints employed by previous HZD-controllers designed for ERNIE. Specifically, joint angles were parameterized as functions of gait progression using 6^{th}

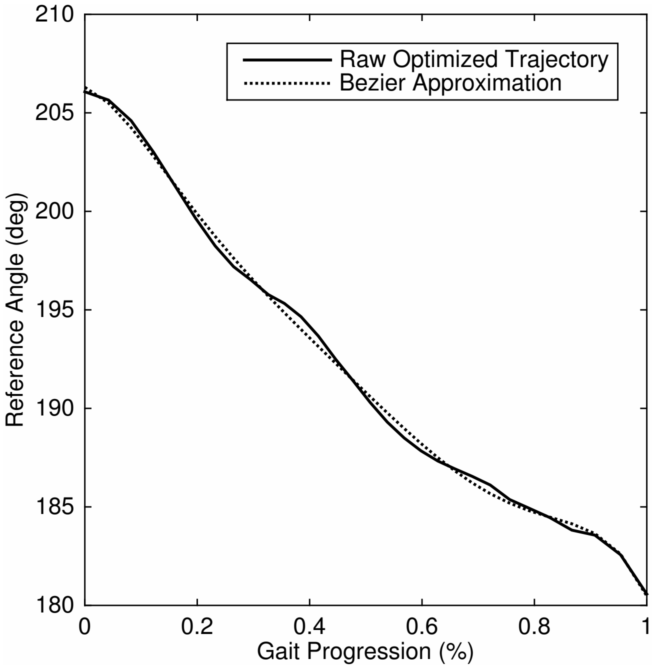

Figure 2: A comparison between a Bézier approximation and the original joint profile generated from collocation

order Bézier polynomials. Gait progression is calculated from the change in angle between the stance foot and the hip over the course of a step. Bézier coefficients were selected to minimize the sum of squared error between the original optimized gaits and the Bézier approximation. Figure 2 shows an example thigh trajectory and its Bézier approximation. While very close to the generated trajectory, the Bézier approximations do not match the source profiles exactly, presenting a possible source of instability in the final gaits. The resulting joint angles were enforced on ERNIE via high gain PD control. While feedback linearization would be a preferable technique, the computational load is problematic for the current ERNIE hardware.

4 Walking Results

For the purposes of this work, periodic walking gaits were generated from 0.4 to 0.7 m/s, matching the experimentally successful range found on ERNIE previously with HZD-based gaits. Step length was left unconstrained, and generally fell between 0.3 and 0.4m. Initial attempts to execute these collocation-based gaits resulted in failure, however their behavior mirrored that of early attempts to get HZD-based gaits working on ERNIE. The primary failure mode consisted of premature swing-foot touchdown caused by a combination of shallow foot approach angle and joint tracking errors. With previous HZD-based gaits, this was solved by including a constraint on the change in swing shank angle in the final 20% of the gait [5]. A new series of gaits was generated incorporating this constraint. This series consisted of 16 gaits with design speeds at 0.2 m/s increments. Of these gaits, 13 walked stably with little or no tuning. Interestingly, a post-facto application of HZD stability analysis showed that only 2 of these gaits were mathematically stable, indicating that the HZD-stability metric is not a necessary condition for experimental success in this system.

5 Conclusion and Future Work

The method of control via virtual constraints which underlies HZD control has been very successful in a wide range of nonlinear control applications from robots to prosthetics [7]. Results shown here indicate that even without mathematical guarantees of stability, the method can often lead to a high degree of experimental success. Moreover, even with errors due to Bézier approximation of the prescribed trajectories, direct trajectory optimization was a valuable tool in this process. This is important as more complicated robots pose a challenge for HZD-based analysis, but may still be amenable to trajectory optimization and virtual constraint control. One example of this need to deal with more complicated systems is the examination of reaction wheel actuation on ERNIE. Virtual constraint control promises to allow the experimental validation of optimization results that showed energy efficiency improvement through minimization of impact losses [2]. ERNIE has been equipped with a reaction wheel system and a virtual constraint framework has been implemented to control the reaction wheel velocity. Preliminary experimental results are promising, with ERNIE already demonstrating short periods of reaction wheel walking.

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