Local Stability of A Flat-Foot Biped with Ankle Compliance under Energy Shaping Control

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1 Introduction

Stability of limit cycle walkers attracts increasing interests in robotics community. Goswami *et al.* demonstrated that a compass biped model, which is identical to a double pendulum, can exhibit periodic and stable limit cycles [1]. Hobbelen *et al.* have studied the effects of ankle actuation on the energy use and disturbance rejection of limit cycle walkers [2]. Most studies on the stability of limit cycle walking have focused on the step-to-step stability, usually evaluated based on the Poincaré sections [3].

However, this mentioned step-to-step method ignored the dynamic behavior of the solution trajectory between Poincaré sections. Norris *et al.* investigated the local stability of a two-dimensional compass-like biped with point feet by analyzing the eigenvalues of the Jacobian of the equations of motion (EoM) [4]. In this paper, we proposed a four-link bipedal walking model with flat feet and ankle compliance, which is more complex compared with the point-foot bipedal walking model presented in [4]. Introducing flat feet enables the study of the local stability in ankle push-off. And the addition of ankle compliance may reveal how to improve the local stability by regulating joint compliance. Furthermore, we conducted simulations with external disturbance and an energy-shaping control method to clarify the relationship between the expansion rate of perturbation and the walking stability. We applied this control method only in the locally unstable regions and effectively reduced the cost of transport. These results may provide a further understanding of local stability of different phases in one step and guide the design of effecient control methods for bipedal robots.

2 Bipedal Model with Flat Feet and Ankle Compliance

We established a four-link model with flat feet and ankle compliance. The model consists of two rigid legs interconnected through a hinge at the hip. Each leg is flexibly connected to a flat foot. The mass of each leg and foot is assumed to be evenly distributed over the corresponding stick. The ankle joints are modeled as passive joints constrained by torsional springs. Each spring reaches the equilibrium position when the foot is orthogonal to the leg.

The local stability of the system $\dot{s} = F(s)$ can be obtained with the eigenvalues of the matrix:

$$D_sF|_{s_{eq}} = \frac{\partial F(s)}{\partial s}|_{s_{eq}} \tag{1}$$

where $s = (\theta_1, \theta_2, \theta_{1f}, \theta_{2f}, p_1, p_2, p_{1f}, p_{2f})'$ is the state vector, $s_{eq}(t)$ is the periodic solution of the limit cycle walking.

The instant evolution of the perturbations can be measured by the local rate of expansion in state space, which is given by:

$$divF(s) = trace(D_sF|_{s_{eq}}) = \sum_{i=1}^{n} \frac{\partial \dot{s}_i}{\partial s_i} = \sum_{i=1}^{n} \lambda_i \tag{2}$$

As $F(s)$ is a conservative system and described in canonical Hamiltonian form, $divF(s)$ always equals zero. However, as mentioned in [4], the direction of underlying periodic orbit should be uncoupled with the other directions in state space, for the perturbation in the direction tangent to the evolution of the periodic orbit has no effects on walking stability. The direction vector of the evolution of the periodic orbit is given by:

$$s_{orb}(t) = \dot{s}/\|\dot{s}\||_{s_{eq}(t)} \tag{3}$$

Then the final expression of the expansion rate of the perturbations in orthogonal directions is:

$$ExpansionRate = trace(D_sF(t)) - \lambda_{\tau\tau} = -\lambda_{\tau\tau} \tag{4}$$

If the expansion rate is positive, the perturbed motion will diverge from the periodic solution, and the walking is locally unstable. In the contrary, negative expansion rate indicates that the perturbations will converge to the periodic solution, and the walking is locally stable.

3 Experimental Results

In this section, we added external disturbance torques in a typical limit cycle walking, and investigated the relationship between the disturbed instances and the resulting walking performance. Fig. 1 shows the expansion rate curve of one step with no disturbances. The disturbances in this study are external torques imposed on every joint with the same magnitude. The magnitude of the disturbances has three levels: $10N \cdot m$, $12N \cdot m$, and $25N \cdot m$. The torques are introduced when the walking cycle proceed to 10%, 25%, 40%, 50%, 65% and 75% of one step respectively, and sustain for $50ms$. The instants when the disturbances are added represent the key points which depict the curve of the expansion rate.

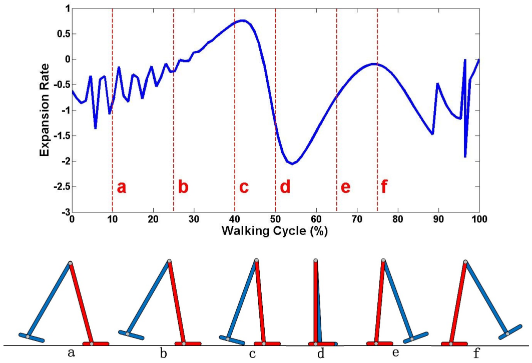

Figure 1: The expansion rate in one stride wdith no disturbances. The dash lines in the above figure indicate the instances when the biped will be disturbed by external torques. The corresponding gait postures are presented below.

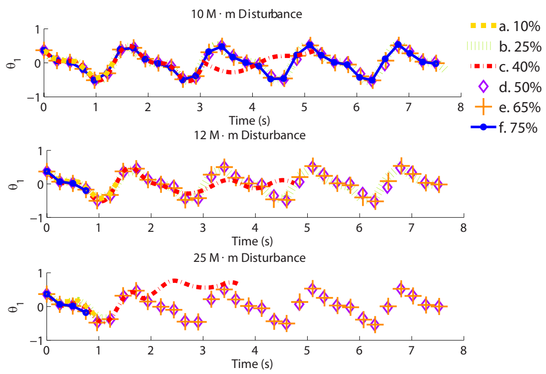

Figure 2: The trajectories of the leg 1 angle θ_1 disturbed by (a) $10N \cdot m$, (b) $12N \cdot m$, and (c) $25N \cdot m$ disturbance torques respectively at different moments in the first walking cycle.

The trajectories of θ_1 are recorded and drawn in Fig. 2 to represent the walking state under each set of disturbance torques. These trajectories show that the perturbation rejection is relative poor when the biped is disturbed at 10%, 40% or 75% of one step (see Fig. 2.a, c and f). In the contrary, the biped can maintain walking under large disturbance torques at 50% and 65% of one step (see Fig. 2.d and e). In conclusion, the perturbation rejection is relatively poor in early swing, before mid-stance and before the swing leg getting to the maximal front position in one step. After mid-stance, the perturbation rejection is relatively strong.

An energy-shaping control method is applied to the biped, the control input is regulated as $u = -\lambda(E - E_d)S^{-1}\dot{s}$, where S is the matrix which maps the control input to each state, E is the actual mechanical energy and E_d is the desired mechanical energy. The control input is introduced to the biped just from early swing to mid-stance and after the swing leg getting to the maximal front position. Compared with full-process energy-shaping control, this local control is more efficient because the biped is only controlled in locally unstable regions. The comparison of the costs of transport with the two control methods is shown in Fig. 3. Successive walking can be realized under $25N \cdot m$ disturbance torques regardless of the disturbed moment (see in Fig. 4).

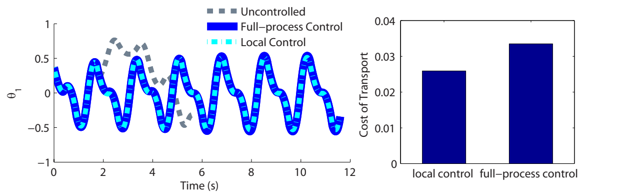

Figure 3: The costs of transport of the bipedal walking with the local and full-process energy-shaping control methods.

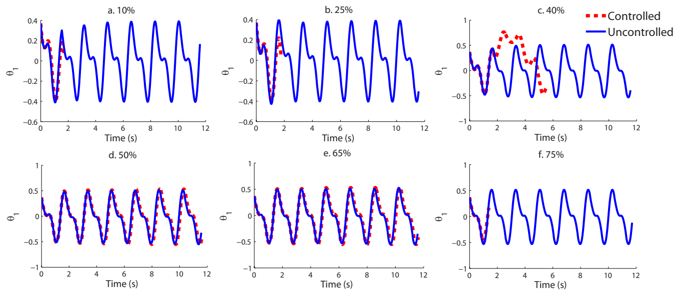

Figure 4: The control performance of the local energy-shaping control method under $25N \cdot m$ disturbances at different moments.

4 Conclusion and Future Works

We studied the local stability of a flat foot biped with ankle compliance. The experimental results in simulations show that the perturbation expansion rate has a strong relationship with the local stability. A local energy-shaping control method with high efficiency is proposed to improve the disturbance rejection.

A focus of future work will be inventing an effective control strategy which can raises the stability of passivity-based bipedal robots by regulating the joint stiffness in specific phases. Furthermore, a limit cycle walking bipedal robot may be developed to testify the experimental results.

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