Walking on a shaky bridge and shaken treadmill: optimal walking motions predict metabolic costs below normal

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1 Introduction

This abstract is a summary of work that has already been published [1], and uses text and figures adapted from this paper. An interesting dynamical phenomenon associated with human locomotion is the large-scale oscillations observed on the opening day of the London Millennium Bridge. Small oscillations of the bridge caused pedestrians to move in a manner that amplified and sustained these oscillations [2]. Motivated by this phenomenon, we introduce a minimal mathematical model of a biped walking on a platform capable of lateral movement. Using numerical optimization, we obtain energy-optimal walking motions for this biped, deriving the periodic body and platform motions that minimize a simple metabolic energy cost.

2 Model Details

We consider a simple biped consisting of a point-mass upper body of mass m and two legs, as shown in figure 1, capable of three-dimensional movement. During each step, only one leg is in contact with the ground, while the other can be swing arbitrarily. The legs are massless for the purposes of model dynamics, i.e. swinging the leg does not affect the motion of the center of mass, but have a mass m_{foot} to calculate the metabolic cost.

The platform, also shown in figure 1, is modelled in two ways:
1. Infinite Inertia Platform: This platform is externally forced to impose a sinusoidal displacement of appropriate frequency and amplitude and behaves like a shaken treadmill.

2. Finite Inertia Platform: This platform is a mass-spring-damper system, with parameters selected to be similar to those of the Millennium Bridge.

The metabolic energy cost model is taken to be the sum of four components: i) the resting metabolic rate, a constant per unit time, ii) Stance work cost, proportional to a linear combination of the positive and negative work performed by the legs during stance phase, iii) Stance force cost, proportional to the integrated leg force during stance, and iv) Swing leg cost, proportional to the work needed to move the swing leg to its next stance position.

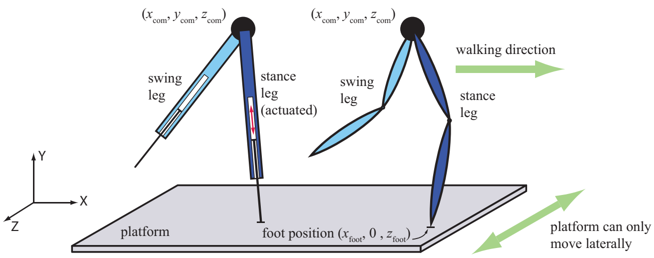

Figure 1: Two eqiuvalent embodiments of the simple biped model walking on a platform. The point-mass upper body can move in all directions, Forward (X), Vertically(Y) and Laterally(Z).The platform can only move laterally.

3 Methods

For either platform model, we parameterize the space of possible walking motions using finitely many unknowns, using initial conditions for the steps and describing the leg forces using piecewise linear functions. We perform numerical optimization with TOMLAB-SNOPT to find the walking motions, periodic over two walking steps that minimize the metabolic cost per unit distance. For the infinite-inertia calculations we constrain the walking motion to be entrained to platform motion and select a walking speed, platform amplitude and platform oscillation-frequency for which to perform the optimization. For the finite-inertia calculations we only specify platform mass, stiffness and damping with the added constraint that platform motion needs to be two-step periodic as well.

4 Results

For both platform models we discovered that the optimal walking motion is always an inverted pendular motion with push-off and heel-strike impulses and with both legs extended to the maximum allowable length. This information was used to simplify the models further and speed up subsequent calculations.

For the infinite-inertia platform (treadmill case) we found that oscillating the platform produced a reduction in the metabolic-energy-cost of the biped compared to a case with no oscillations (normal walking), as seen in figure 2. Increasing the amplitude of oscillations reduced this cost even fur-

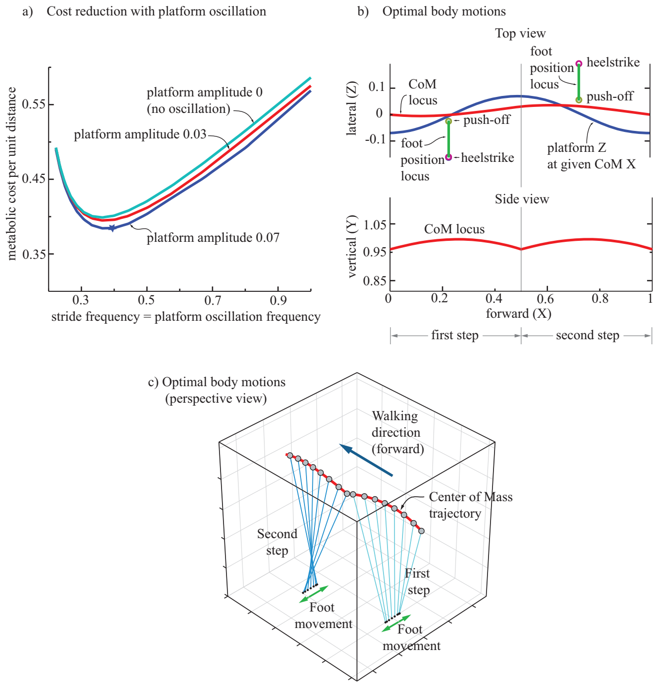

Figure 2: Energy-optimal walking on an oscillating infinite inertia platform. a) Cost reduction with platform oscillation, b)optimal body motions and c) optimal body motions (perspective view)

ther. We find that by appropriately synchronizing the body motion with the platform motion the model can reduce the magnitudes of heel-strike and push-off impulses, thereby reducing the positive and negative stance leg work produced by these impulses.

For the finite-inertia platform (bridge case) we found a critical number of pedestrians above which an energy-optimal motion required platform oscillations. As seen in figure 3, this critical number of pedestrians depends on both the platform damping and the platform stiffness. Similar to the treadmill case, there is reduction in stance work required of the legs when the platform oscillates. However, the presence of damping means that the pedestrian must do extra work to sustain these platform oscillations. These competing costs are the source of the critical pedestrian count.

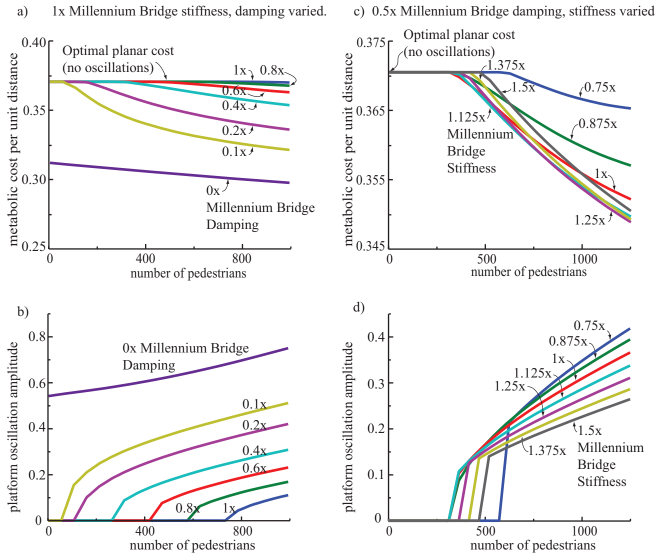

Figure 3: Energy-optimal walking on the finite inertia platform shows platform oscillations reduced metabolic cost when there are sufficiently many pedestrians. a) Metabolic energy per unit distance and b) Platform oscillation amplitude versus number of pedestrians for the exact Millennium Bridge stiffness and different multiples of Millennium Bridge damping. c) Metabolic energy per unit distance and d) Platform oscillation amplitude versus number of pedestrians for stiffness in multiples of Millennium Bridge stiffness and 0.5X Millennium Bridge damping

References

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