Stability of Planar Compass Gait Walking with Series Elastic Ankle Actuation

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1 Introduction

Passive dynamic walking models capture the natural dynamics of stable human-like walking. The passive compass gait (PCG) model, consisting of a point mass and two rigid legs, is among the simplest of such models. The fully passive nature of these models, however, necessitates a sloped ground to recover the energy lost during the ground collisions [1]. A variety of methods have been proposed to eliminate this requirement through different actuation methods. Among these are impulsive energy injection after foot collision, torque actuation on the hip, active ankle joints or tunable compliance in the leg [2, 3]. In this study, we propose a simple model to investigate how series elastic actuation (SEA) at the ankle can be used to achieve stable walking on level ground. The structure we propose is designed to behave in a similar fashion to how humans utilize toe push-off prior to leg liftoff, and is intended for eventual use within a lower-body robotic orthosis.

We begin by deriving the hybrid equations of motion, resulting in a numerically computed single-stride return map. We then identify fixed points of this system and characterize their stability, showing that this model exhibits asymptotically stable walking on flat ground. We also study the dependence of limit cycles and their stability on system parameters.

Our motivation for studying actuated ankles for dynamic walking comes from our longer term goal of implementing efficient powered lower-extremity robotic orthoses for individuals who have lost function in their lower extremities. Existing work to this end focuses only on restoring knee and hip joint functionality. However, excluding ankle joint actuation is likely to substantially reduce energetic efficiency and lead to unnatural walking patterns. In light of these observations, our study seeks to understand the impact of using SEA for the ankle on walking dynamics, towards eventual integration with powered robotic lower-body orthoses.

2 The Ankle Actuated Compass Gait Model

The planar Ankle Actuated Compass Gait (AACG) Model we focus on in this study is illustrated in Fig. 1. It consists of a body mass $M = 1kg$, to which two legs of length $l = 1m$ with small mid-length masses $m = 0.01kg$ are attached, all constrained to the saggital plane. Walking with this model has a similar structure to the PCG model, with an important difference in how transfer of support from one leg to the other is realized.

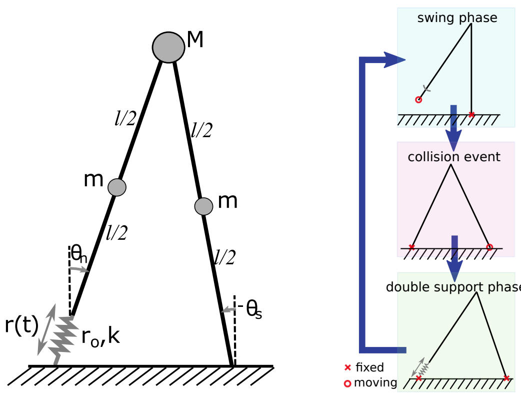

Figure 1: The Ankle-Actuated Compass Gait (AACG) model (left) and phases of its locomotion (right).

During the single support phase, the system pivots around its support toe, assuming that "toe-stubbing" collisions are avoided. Continuous dynamics for this phase are identical to the PCG model and can be found in existing literature. When the swing leg collides with the ground, however, we deviate from the PCG model and switch to a "thrust" phase with an active ankle. Since the stance toe maintains contact with the ground, angular momentum may not be conserved, requiring explicit computation of impulsive forces. To this end, we define the configuration of the three degree of freedom system dynamics during the collision event as

$$q_c := [\theta_s, \theta_n, r]^T \; . \tag{1}$$

The dynamics of this phase under impulsive forces and collision constraints yields a collision map

$$\dot{\mathbf{q}}_c^+ = \mathbf{H}_c(\mathbf{q}_c^-)\dot{\mathbf{q}}_c^- \; , \tag{2}$$

where $\dot{\mathbf{q}}_c^-$ and $\dot{\mathbf{q}}_c^+$ denote system velocities before and after the collision, respectively.

Following the collision, we assume that a "pre-compressed" ankle spring in series with the previous stance leg is released. This results in a double support phase where this ankle spring provides active thrust from the fixed back toe, with the front leg pivoting freely around its corresponding fixed toe. This continues until the ankle spring is fully extended, at which point the system continues with the next single stance phase.

3 Simulation Studies

We use Poincaré methods to analyze the presence and stability of limit cycles for the AACG model. We define the Poincaré section as the vertical state of the stance leg within single with $\theta_s = 0$ and $\dot{\theta}_n > 0$. This corresponds to the highest point of the body mass during single stance, which we call here the *apex point*. This results in the apex to apex return map

$$\mathbf{x}_{i+1} = \mathbf{G}(\mathbf{x}_i), \tag{3}$$

where $\mathbf{x} := [\dot{\theta}_s, \theta_n, \dot{\theta}_n]^T$ defines the Poincaré section coordinates. In this study, we numerically compute this map and its Jacobian to identify limit cycles and their stability.

In order to present a more complete picture of system behavior, we also investigate the dependence of limit cycles (as fixed points of the apex return map) and their stability on spring parameters. Fig. 2 shows our preliminary results for fixed points of the single stride return map as a function of the amount of spring pre-compression. We have also found period two and above limit cycles, but we leave a more careful study of these different behaviors for future work.

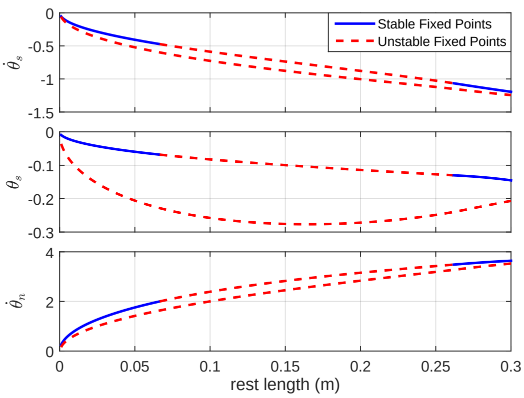

Figure 2: Fixed points of the AACG model with $k = 100N/m$ as a function of ankle pre-compression. Solid and dashed regions show stable and unstable fixed points, respectively.

Finally, Figs. 3 and 4 show all three eigenvalues associated with the two separate limit cycles shown in Fig. 2 as a function of the ankle spring rest length. One of the eigenvalues is always zero, corresponding to the dependence of swing leg position and velocity prior to the collision. As the rest length increases, one of the other eigenvalues also converges to zero for both fixed points. In summary, our results show that the AACG model exhibits stable limit cycles for a range of spring rest length values. This suggests that the use of series elastic actuation for the ankle joint of a walking platform is feasible with promising stability properties.

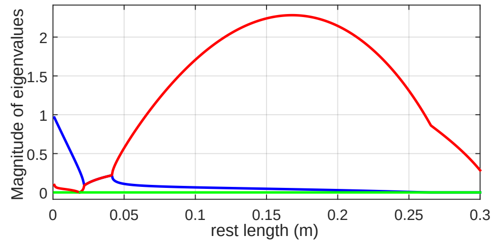

Figure 3: Eigenvalues of the linearized apex return map for the first fixed point as a function of the ankle spring rest length r_0 with $k = 100N/m$.

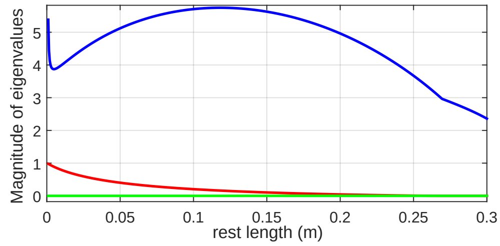

Figure 4: Eigenvalues of the linearized apex return map for the second fixed point as a function of the ankle spring rest length r_0 with $k = 100N/m$.

4 Conclusion

In this study, we focused on a dynamic walking model with active ankle actuation through series elastic actuation. Our model summarizes the action of the normally rotary ankle joint with a simplified prismatic joint. The spring situated in this joint is activated immediately following the collision of the swing leg with the ground, modeling toe push-off. Equations of motion associated with all locomotory phases for this model were implemented in Matlab, yielding a numerically computed Poincaré return map and its Jacobian. Fixed points of this map were used to identify limit cycles of this system and their stability. Our preliminary results show that the Ankle Actuated Compass Gait model we described in this study exhibits locally asymptotically stable limit cycles corresponding to feasible, sustained walking on flat ground. We plan to extend our analysis further through more careful studies of parametric changes in limit cycles, including solutions that are period two or higher.

References

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