Wrong and Useful: Metrics to Assess Simple Walking Models

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1 Introduction & Motivation

Simple mechanical models of legged locomotion are useful in legged robot control [4], prosthesis design [1], human balance assessment [2], and examining goals in natural gait [6]. Another approach to studying gait involves much more detailed modelling of the system [5]. There is a tradeoff in the accuracy and scope of the model versus the ability to interpret and generalize results. Complex models may require more parameters to tune, and high dimensionality may pose a problem for realt-time trajectory optimization methods. Choosing a model to study a specific question about a walking system performing a given task therefore requires engineering judgement about what aspects of the system and task are relevant.

We are curious about which aspects of walking dynamics are most essential. Many walking models include a falling mode. The limitation on ground reaction forces inherent in a legged system severely restrict what motions are possible. One useful abstraction of this limitation is to consider a model of walking with a point foot, for example a linearized inverted pendulum model (LIPM). The two dimensional LIPM exhibits the falling mode that likely dominates stability in a wide range of legged locomotion. However, the amount that this falling mode dominates a particular walking system, natural or artificial, undergoing a particular motion, is not well quantified. In addition, we believe that the dynamics of the swing leg, and not just the falling of the center of mass, may represent an important aspect of gait, particularly at higher walking speeds.

In addition, we wish to quantify both how "useful" and how "wrong" a model is. Here we consider a model's usefulness to be related to its ability to predict the future evolution of the legged system, in the absence of any other models or information. A model's wrongness is related to how damaging the assumption of that model is to the prediction of the legged system. That is, even if other models or information are available to assist in predicting the system's evolution, forcing the prediction to be compatible with a very wrong model would necessarily introduce innacuracies.

2 Our Approach

We seek to quantitatively compare the predictive powers of simple models of walking for a given task. As a simplification, we will consider the ability of a simple model to predict the state derivative of the complex model at a given state. This prediction is therefore valid only locally in time and state space. The system to be studied will be a passive kneed walking model. We will compare how much of the dynamics of the kneed walker can be captured by an inverted pendulum model (LIPM) (with no swing leg mass) versus a compass walker (with swing leg mass). At states throughout a gait-like motion of the kneed walker, we will compare the ability of each of the two simpler models to predict the future (local) behavior of the kneed walker, $\dot{x}_c$.

Two methods are used to predict the kneed walker segment angle velocities using each model. Briefly, both methods involve projecting the kneed walker state, x_c, into the state space of the simple system x_s, then calculating the derivative, using using a transformation T_s and equations of motion:

$$x_s = T_s(x_c) \quad \text{(1a)} \qquad \dot{x}_s = f_s(x_s) \quad \text{(1b)},$$

For the LIPM, T_s matches the total center of mass between the models. For the compass walker, T_s matches the hip location and angle of the mass of the swing leg (Figure 1). The local prediction for the simple model ($\dot{x}_s$) does not uniquely predict the behavior of the complex model ($\dot{x}_c$) since for any given $x_{s,n}$, a family of x_c satisfy the relationship (1a). Here we use two different methods to predict the complex behavior given

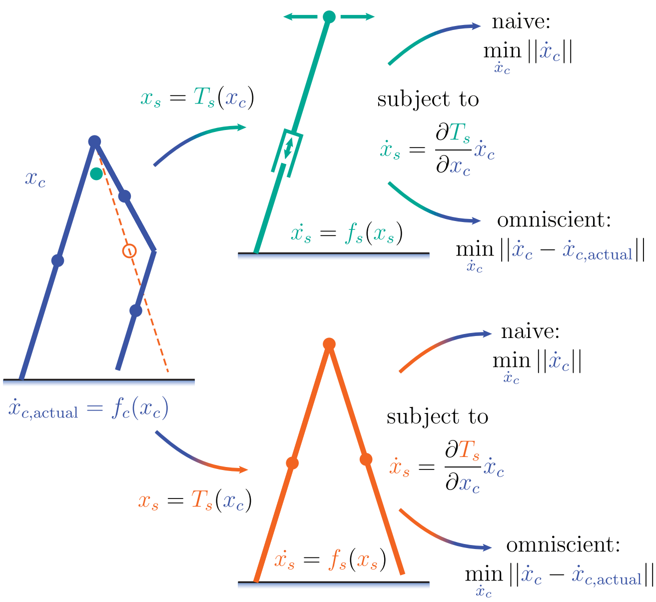

Figure 1: The state if the kneed walking model under study (left), is transformed to two simpler models, the linear inverted pendulum model (top), and the compass walker (bottom). The state derivative for each simple model is calculated using the respective model's equations of motion. This state derivative is then transformed back into the space of the keed walker using two different methods, both constrained to be consistent with the simple model.

$\dot{x}_s$. The first, the naive prediction, calculates the $\dot{x}_c$ with minimal norm, while satisfying (1a). The second, the omniscient prediction, assumes that the acceleration of the complex system is known, and finds the $\dot{x}_c$ that is closest to $\dot{x}_{c,\text{actual}}$ while still satisfying (1a).

A naive prediction close to the actual state derivative indicates that the simple model can represent the kneed walker well with no other information (and therefore may be quite useful). An omniscient prediction very different from the actual state derivative indicates that the simple model introduces significant inconsistency with the dynamics of the kneed walker (and is therefore probably quite wrong).

3 Preliminary Results

The kneed walking model was initialized in a state with the knee bent and swinging forward (Figure 1), then integrated forward in time passively for 0.3 nondimensional time units. The $\dot{x}_c$ was predicted at each time step using both the naive and omniscient methods, for each model (Figure 2). The compass model provides a reasonable prediction of the stance leg angular velocity using both the naive and omniscient methods. However, the prediction of the swing thigh and shank velocities is substantially better using the omniscient method than the naive. The linear inverted pendulum model predicts the stance leg and swing thigh velocities poorly using both methods, and the swing shank velocity is only predicted well using the omniscient method.

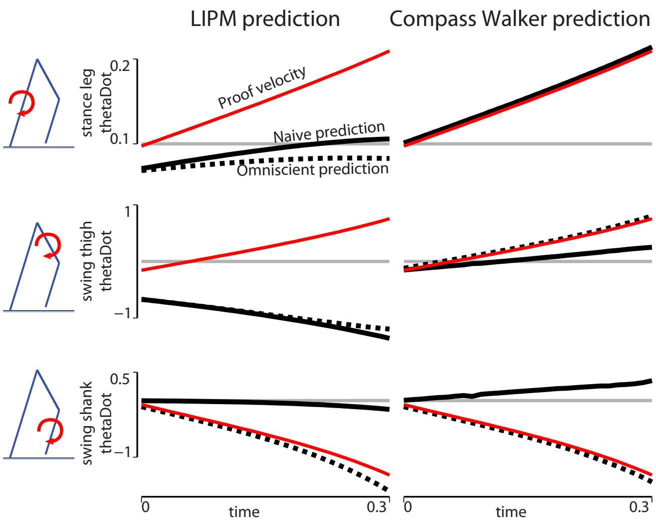

Figure 2: Comparison of the ability of two simple models to predict velocities for a planar kneed walker through a range of states. Each plot shows the velocity of a joint of the kneed walker based on the naive prediction of the model (that is, velocities that are consistent with the transformation T between the kneed walker and the simple model, and minimal in a least squares sense). Also shown are the omniscient predictions, which are consistent with T and are minimally different from the actual velocities, in a least squares sense.

4 Discussion

The LIPM generally provided a poor prediction of the system's evolution for the stance leg and swing thigh. However, using the omniscient method, a better fit was found for the swing shank velocity. This suggests that, in the states considered, the swing shank had little effect on the projected LIPM state, so the optimization process was able to achieve better prediction. Therefore, while the LIPM was not useful in predicting the swing shank velocity, it was also not so wrong as to limit the ability to predict it given sufficient additional information. The compass walker was found to be useful to predict the stance leg velocity (the naive prediction is good), and also is not very wrong for all the states (the omniscient prediction is good for all three states). This is likely due to the fact that for any given motion of the compass model's swing leg, a combination of thigh and shank motions can be found which largely replicate the kinematics of the compass model.

One limitation of the current work is the focus on the predicted velocities, as opposed to accelerations. We plan to analyze accelerations, as well as more complete walking gaits, including hybrid dynamics. We will extend this analysis to consider the effect of impacts.

Other methods could quantify the performance of simple models. For example, linearizing the complex system's dynamics and performing a singular value decomposition can lead to a lower dimensional system [3]. This obviates the need to design a simple system, as well as the transformation function from the complex to state to the simple state. However, one advantage in designing a simple system is that intuition can be used to guide the process. Also, exploration of different simple models may help build intuition and answer questions regarding the mechanisms involved in walking or other tasks. We predict for example that adding a swing leg to a model may make more sense when the system is walking faster, as the inertia of the swing leg may start to play a larger role. More generally, we propose this work to contribute to a discussion of why we choose a given model for a given task.

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