Stabilization of Energy Optimal Series Elastic Gait

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1 Introduction

Series elastic actuation has often been proposed as a mechanism for energy recovery in legged robots [1]. By storing and releasing energy in springs similarly to how tendons function in mammals, it is expected that the energetic efficiency of a robot can be significantly improved. This thesis has been demonstrated in simulation for a variety of systems, ranging from more conceptual spring loaded inverted pendulums to full quadrupeds. In hardware, however, demonstrations of efficiency have been limited to simple, open-loop systems, and have yet to be shown on more versatile robots [2].

A major reason for this disconnect between simulation and hardware implementation lies in the challenge of stabilizing under-actuated limit cycles. Traditional control for periodic tasks takes the form of reference tracking, in which the controller stabilizes a time varying operating point that moves along the desired trajectory. However, these methods rely on having full control authority in a neighborhood of the trajectory. When this authority is lost (i.e. in under-actuated systems), the controller breaks down. This breakdown is due to the fact that under-actuated systems often require complex trajectories to arrive at a point in state-space.

What is needed in lieu of reference tracking is a controller that works to stabilize the entire limit cycle, as opposed to a moving point on the orbit. One approach towards this is to define a set of simple controllers around the limit cycle and associate regions of state-space with each controller. Control optimization methods can then be used to ensure stability. Examples of this approach include differential dynamic programming [3] or transverse LQR [4]. Currently, these methods suffer from small regions of attraction that are caused by the limited accuracy of the locally quadratic models. To improve the robustness of such controllers, Erez et al. [5] have proposed an infinite horizon model predictive control (IHMPC) scheme.

The basic idea of IHMPC is to use differential dynamic programming (DDP) to find a model-optimal *n*-step controller for the current robot state. The terminal cost for this DDPformulation is a locally quadratic expansion of the cost-to-go function around the limit cycle. The fast convergence rate of DDP allows us to perform this optimization at rates on the order of ≈ 10 Hz, which in turn means that the controller can respond to disturbances and model inaccuracies in real-time. Additionally, since the controller is continually pushing the system towards the optimal limit cycle, we expect a nearoptimal behavior in the limit of small disturbance. In their work, Erez et al. have implement IHMPC in the simulation of a three link hopping monoped. They demonstrated robustness to both disturbance and model inaccuracy. We are currently working towards scaling these results to a 5-link series elastic bipedal walker and implementing the method on our robot RAM*one* (Fig. 1).



Figure 1: The planar 5-link biped 'RAM*one*' will provide the experimental hardware platform for the proposed study. The robot's legs have high-compliance series elastic actuation with un-actuated feet.

2 Current Results

We started exploring this approach with the model of a simple one-DOF series elastic hopper (Fig. 2). The equations of motion are as follows:

$$\frac{d}{dt} \begin{bmatrix} y \\ \dot{y} \\ d \end{bmatrix} = \begin{bmatrix} y \\ \frac{k_{spr}}{m} \max(0, d+l_0-y) - \dot{y}\eta - g \\ u \end{bmatrix}$$

Where y is the hopper height, d is the actuator displacement, u is our control input, l_0 is the unstretched spring length, η is the damping constant, and g is gravitational acceleration. For the optimization, we define our cost function to be:

$$\ell(x) = \alpha \left(1 - e^{-\tau (y - y_{des})^2} \right) + \beta u^2 + \gamma \left(d - \frac{d_{max}}{2} \right)^2$$

This can be seen as a combination of penalties on: 1) deviation from desired height, 2) control input, and 3) deviation from the middle of the actuator stroke. The massless foot of the hopper allows for a non-hybrid dynamic formulation, and the limited number of dimensions allows us to effectively compute and visualize results.

Using a multiple shooting framework (MUSCOD-II [6]), we found an optimal periodic motion for this model. The controller for this trajectory is open loop, and the resulting behaviour is an unstable limit cycle. We fit a discrete set of quadratic models of the cost-to-go around this limit cycle using nonlinear least squares, and use this value estimation to generate a full state-space control scheme (Fig. 2). We are currently in the process of implementing the *n*-step look-ahead optimal controller to broaden the region of validity of this local model.



Figure 2: 1 dimensional series elastic hopper model used to test our model predictive controller. The control input is the actuator velocity, and the hopper body experiences viscous damping.

3 Expected Results

After successful implementation of this controller on the simple model, we plan to move on to a detailed model of RAMone and eventually to the actual robot. By the time of the conference, we expect to have results in simulation and ideally some preliminary hardware results. A major conceptual challenge in moving towards this hardware, is the hybrid nature of the real dynamics of the robot. With mass in the legs and feet, we expect contact collisions and (quasi) discontinuities in velocities at impact. Since DDP relies heavily on continuous dynamics, we will investigate methods for smoothing these transitions. This could be achieved, for example, through stochasticity [7], or soft contact models. Expected results will consist of detailed explorations into the stability, robustness, and efficiency of this control scheme. Stability will be characterized by estimates of the region of attraction of the controlled system (using either Monte Carlo or sumsof-squares methods). Efficiency will be characterized by simulated cost of transport, robustness will be tested with explicit introduction of model inaccuracies. We have also begun in-



Figure 3: Optimal limit cycle and locally optimal control policy for the simple hopper.

vestigating the effects of update rate on the controller performance, and intend to include these results in the poster.

References

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