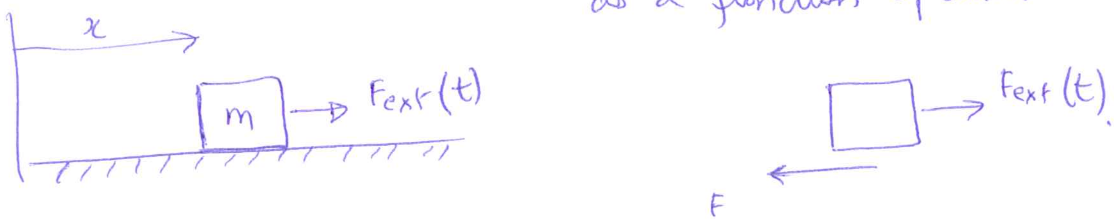
SIMULATING COULOMB FRICTION WITH EVENT DETECTION

SYSTEM I : Mass on a frictional surface with an external force $F_{ext}(t)$ as a function of time.



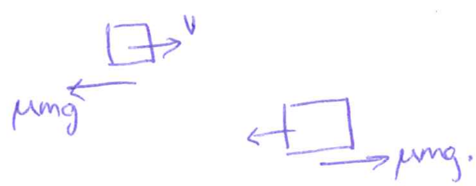
F - friction force . Coulomb friction.

When the mass is sliding, the friction force magnitude is μmg (independent of the relative sliding velocity). And the friction is opposite in direction to the relative velocity.

Say $v = \dot{x}$ = relative sliding velocity between the 2 surfaces.

When $v > 0$, $F = \mu mg$.

When $v < 0$, $F = -\mu mg$.



When the mass is not sliding, the friction force F is bounded by $\mp \mu mg$. ie, $-\mu mg \leq F \leq \mu mg$.

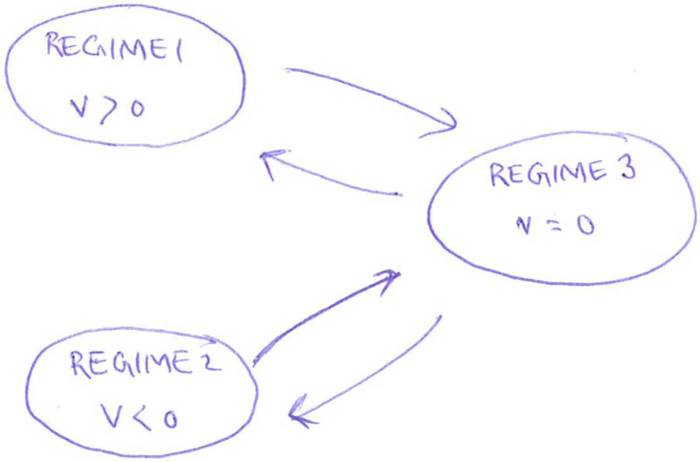
That is, when the mass has $v = 0$, the mass continues to have $v = 0$ as long as $-\mu mg \leq F_{ext} \leq \mu mg$.

So to simulate <u>system 1</u>, we can consider the following 3 regimes:

$V > 0$:	$m\ddot{x} = F_{ext} - \mu mg$	Regime 1
$V < 0$:	$m\ddot{x} = F_{ext} + \mu mg$	Regime 2
$V = 0$:	$m\ddot{x} = 0$	Regime 3

<u>How do you switch from one regime to another?</u>

The following transitions are possible:



<u>REGIME 1 to REGIME 3</u> : V becomes zero, while decreasing.
(value = V, direction = −1)

<u>REGIME 3 to REGIME 1</u> : $F_{ext}(t)$ exceeds μmg so the mass starts sliding.
(value = $F_{ext}(t) - \mu mg$, direction = +1).

REGIME2 to REGIME3 : v becomes zero, while increasing
($value = v$, $direction = +1$).

REGIME3 to REGIME2 : $F_{ext}(t)$ becomes less than (more negative than) $-\mu mg$, so the mass starts sliding with $v<0$.
($value = F_{ext}(t) + \mu mg$, $direction = -1$).

Note that while REGIME1 and REGIME2 have only one event (for each of them), REGIME3 has 2 possible events - resulting in transfer to REGIME1 or REGIME2.

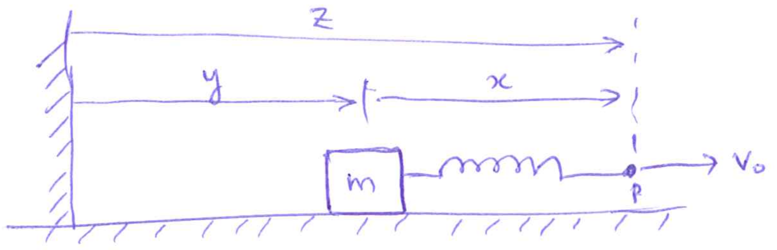
This is possible in MATLAB.
For the event function for REGIME2, we just need Vector functions for value, isterminal, direction, etc.

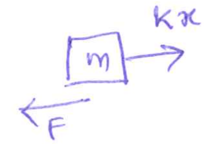
$$value = \begin{bmatrix} F_{ext}(t) - \mu mg; \\ F_{ext}(t) + \mu mg \end{bmatrix}.$$

$$isterminal = [1;1].$$

$$direction = \begin{bmatrix} +1; \\ -1 \end{bmatrix}.$$

SYSTEM 2 : Mass on a frictional surface, connected to a spring k, the spring being pulled at constant velocity V_0.





General equation : $m\ddot{y} = -F + kx$

$$\left.\begin{aligned} y + x &= z \\ \dot{y} + \dot{x} &= V_0 = \dot{z} \\ \ddot{y} + \ddot{x} &= 0 = \ddot{z} \end{aligned}\right\} \Rightarrow \ddot{y} = -\ddot{x}$$

So $\boxed{m\ddot{x} + kx - F = 0}$

where F is the friction force.

Note that F depends directly on $\dot{y}$, which is the relative velocity between mass m and the surface.

F is given by Coulomb friction.

$$\left.\begin{aligned} F &= \mu mg \quad \text{if} \quad \dot{y} > 0 \\ F &= \mu mg \quad \text{if} \quad \dot{y} < 0 \end{aligned}\right\} \boxed{\text{Sliding}}$$

and $\boxed{\text{No sliding}}$ if $\dot{y} = 0$ and

$$-\mu mg \leq F_{ext} \leq \mu mg .$$

The external force that friction needs to balance here is 'kx'.

So, **no sliding**

if $\dot{y} = 0$

as long as $-\mu mg \leq kx \leq \mu mg$.

How to write the ODEs?

REGIME 1 $\quad \dot{y} > 0 \quad$ SLIDING FORWARD

ODE. $\quad m\ddot{x} + kx - \mu mg = 0$

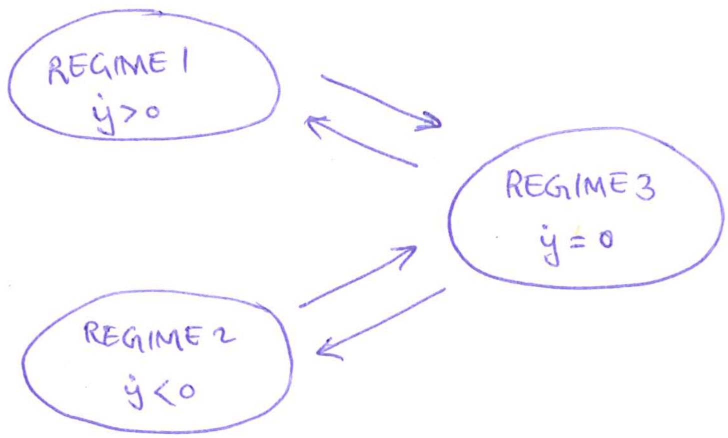
REGIME 2 $\quad \dot{y} < 0 \quad$ SLIDING BACKWARD

ODE. $\quad m\ddot{x} + kx + \mu mg = 0$

REGIME 3 $\quad \dot{y} = 0 \quad$ NOT SLIDING.

ODE $\quad \left.\begin{array}{l} \dot{y} = 0 \\ \ddot{y} + \ddot{x} = 0 \end{array}\right] \Rightarrow \left.\begin{array}{l} \dot{x} = V_0 \\ \ddot{x} = 0 \end{array}\right]$ ← These equations are just a restatement of the fact that this is the "NO SLIDING" regime.

What are the various events?



REGIME 1 to REGIME 3

value $= \dot{y} = V_0 - \dot{x}$

direction $= -1$.

REGIME 3 to REGIME 1

value $= kx - \mu mg$

direction $= +1$.

REGIME 2 to REGIME 3

value $= \dot{y} = V_0 - \dot{x}$

direction $= +1$.

REGIME 3 to REGIME 2

Value $= kx + \mu mg$

direction $= -1$.

So again, when integrating the REGIME 3 equations, one needs to keep track of 2 possible events (although for this problem, REGIME 2 is probably irrelevant) the event function for REGIME 2 is like:

$$\text{value} = \begin{bmatrix} kx - \mu m g \\ kx + \mu m g \end{bmatrix}.$$

$$\text{isterminal} = [1;\ 1].$$

$$\text{direction} = [+1;\ -1].$$

Again: if all you want to simulate is the system 2 with $V_0 > 0$ and constant, REGIME2 will never occur, so in principle, it would be sufficient to consider only 2 regimes namely REGIME1 and REGIME3.

Aside: Sometimes, people assume different friction coefficients for the no-sliding (μ_s) and the sliding case (μ_K), with $\mu_K < \mu_s$, to model the so-called "Stribeck effect".

Here we have assumed $\mu_K = \mu_s = \mu$ for simplicity.