

Near-Simultaneous Footfalls Lend Stability to Multi-Legged Gaits

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1 Motivation and State-of-the-Art

Parsimonious dynamical models for legged locomotion are piecewise-defined. The state flows in continuous-time according to an ordinary differential equation (ODE) until a *touchdown* or *liftoff* event occurs, triggering an instantaneous reset at a discrete time instant [1]. We've shown that models for periodic gaits with footfalls isolated in time (e.g. *bipedal walk* or *run*, *quadrupedal walk* or *gallop*) reduce to classical dynamical systems—smooth ODEs on smooth manifolds [2]. However, footfalls are not isolated for general behaviors.

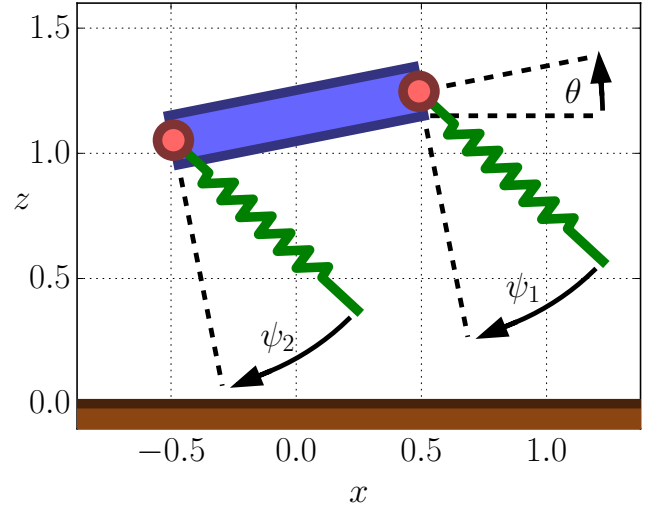
During other typical gaits (*pace*, *trot*, *hop*, *pronk*, *alternating tripod*), transitions between gaits, or aperiodic maneuvers, footfall events can occur at arbitrary times with respect to one another. From a modeling perspective, this can introduce ambiguities or inconsistencies. In models with rigid limbs, for instance, others [3, 4] have observed that the dynamical system flow is generically discontinuous near simultaneous-footfall events. This implies that the model's behavior has unbounded sensitivity with respect to initial conditions. Though potentially relevant for rigidly-constructed artifacts, such singular model behavior provides a poor prediction for the dynamics of physical locomotors—biological or robotic—with viscoelastic limbs.

2 Our Approach and Results

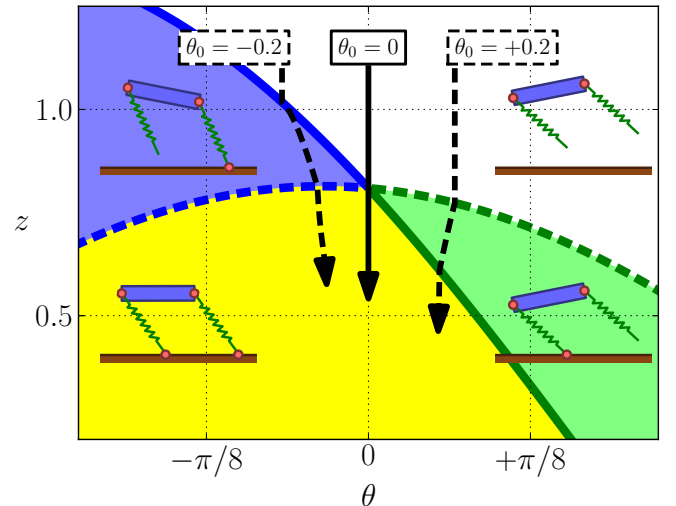
We study models for locomotion with *viscoelastic* limbs as in Fig. 1a. The flow of such systems is generated by a vector field that is *event-selected C^r* [5, Def. 1], i.e. smooth except along a finite number of surfaces of discontinuity as in Fig. 1b. This flow is continuous but not classically differentiable; however, we show in [5] that it admits a strong first-order approximation (a *Bouligand derivative* [6]) that enables analytical characterization of contractive effects arising near simultaneous-footfall events.

We show that, in contrast to the case with rigid limbs, the flow of a multi-legged model with viscoelastic limbs is generally *contractive* near simultaneous-footfall events [5]. This implies that near-simultaneous touchdown lends stability to multi-legged gaits and maneuvers. For periodic gaits wherein limbs partition into groupings that touch down nearly simultaneously (*pace*, *trot*, *pronk*, *alternating tripod*), this provides a first-order synchronizing effect between limbs within a group. Furthermore, whenever a group of limbs undergoes near-simultaneous touchdown, they introduce a first-order

stabilizing effect to one or more body posture degrees-of-freedom (DOF). In abstracted models of locomotion, these effects alone can stabilize a gait. In a more general setting, these effects can be exploited to lend stability to a behavior.



(a) Illustration of multi-legged model with three body degrees-of-freedom (position (x, z) and pitch θ) and two leg angle states ψ_1, ψ_2 . Limbs attach to the body and ground through pin joints.



(b) Trajectories undergoing near-simultaneous footfall (initialized with pitch angle $\theta_0 = \pm 0.2$) contract toward simultaneous-footfall trajectory ($\theta_0 = 0$).

Figure 1: In a viscoelastic-limb model (a), discontinuous dynamics near a simultaneous-footfall event lend stability to body pitch (θ) and synchronicity to limbs (ψ_1, ψ_2).

Notes and Support

We presented an earlier version of these results at Dynamic Walking 2013 [7]. In the intervening years, we made substantial progress on both the underlying theory and the practical relevance for design and analysis of multi-legged locomotion.

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