# Perturbation rejection in bipedal walking models: energetic optimality predicts human-like responses

Barrett C. Clark, Yang Wang and Manoj Srinivasan\* \* The Ohio State University, Columbus, USA clark.1872@osu.edu, wang.1513@osu.edu, srinivasan.88@osu.edu

#### 1. Introduction

Humans are capable of a wide range of gaits, from the common (e.g., walking and running) to the less common (e.g., hopping, skipping, sideways walking). During any of these forms of locomotion, humans are able to walk stably, rejecting perturbations. Unfortunately, we still do not have a good understanding of the human control structure that allows us to perform at such a high level. Numerous papers have been written hypothesizing a control structure and implementing it (e.g., [1, 2, 3]) in simulation or on robots. While some of these controllers work, they seem incapable of the same performance as a human. Other papers have been written on human walking experiments where subjects were perturbed during the gait cycle and their responses were observed (e.g., [4]). These papers offer valuable insight into how humans react and allow us to document general trends in behavior, but they frequently do not give a full picture of how humans control their reactions.

Here, our goal is to demonstrate that metabolic energy optimality can be used not only to predict qualitative trends in steady-state human motion (e.g., [5, 6]), but also to predict human reactions to perturbations during walking. We present two simple models that can predict qualitative aspects of human walking during steady-state motion and calculate their optimal trajectories back to this motion after a variety of perturbations. We then compare their responses to a data-driven model of human walking and show that the energetically optimal trajectories back to steady-state for these simple models is similar to the response from the data-driven model.

## 2. Methods

The central calculation is illustrated in Figure 1a. The periodic walking motion for a bipedal model is first calculated based on a minimization of a simple metabolic energy cost. Then, for various perturbations off the optimal periodic motion, the optimal transient path back to the steady-state motion is found.

**Three-dimensional bipedal models.** For this study, we use two bipedal models to calculate optimal trajectories for walking. The first model, shown in Figure 1b, consists of a pointmass upper body [6]. It is capable of moving the upper body and foot anywhere in three dimensions. The massless stance leg does work on the upper body by applying a force F. By assuming the swing leg's mass to be significantly less than



Figure 1: a) A theoretical steady-state path (blue) with a transient path returning to it (red) b) The point-mass model c) The rigid body model.

that of the upper body, we neglect it in the equations of motion. The second model, shown in Figure 1c, consists of a single rigid body instead of a point-mass. The same massless legs attach to the body at different points, yielding a non-zero hip width. They can produce a moment about the hip as well as force F. The models are the same except for these differences.

**Cost function to be minimized.** We use a simplified metabolic cost model. This model has three terms: a workbased stance cost, a swing cost, and a resting cost.

Numerical optimization. Using numerical optimization, we solve for the initial conditions, foot position on the ground plane, and leg forces (and hip moments when relevant) as a function of time that yield a walking motion that meets the desired constraints and minimizes the metabolic cost model. We use the nonlinear-optimizer SNOPT to solve all optimizations [7]. This solver enforces equality constraints demanding continuity of body state between adjacent steps, bounds on leg forces, hip moments, and leg length. In order to find the optimal response to a perturbation, we performed two different types of optimization. The first optimization found the the optimal periodic motion, enforcing periodicity of state variables over N = 3 steps. The second optimization calculated the optimal transient back to the periodic motion from a given

perturbed state, enforcing that after N = 3 steps, the state variables were exactly the same as in the periodic motion.

**Data-derived model of human walking.** To compare our optimal perturbation responses, we use a data-derived model of human walking dynamics. These are linear models for deviations from the mean walking motion. We derive these linear models from normal human walking, which is nearly, but not exactly, periodic; we exploit this aperiodicity, assuming this arises from various noise. From such noise-driven motions, we determine the linear mapping that uses state deviations from the mean periodic motion at one phase of the periodic walking motion to predict state deviations at another phase of the periodic walking motion, as in [8].

#### 3. Results

We compared various aspects of transients back to steadystate including, for instance, the trajectory of the center of mass and the position of the foot placements. Here, we just discuss the foot position dynamics comparisons. Figure 2 compares the foot positions for sideways velocity and position perturbations of the center of mass obtained from both optimization and the data-derived linear model. As has been shown in human perturbation experiments, subjects tend to adjust their stance foot in the direction of the perturbation, especially for sideways perturbations [8]. In other words, if someone is pushed to the left during a right stance phase, she will move her next stance foot to the left. This feature is predicted by both models for perturbations in the opposite direction of the stance foot, as is shown in Figures 2a and c. The point-mass model predicts a symmetric response to positive and negative perturbations, as one would expect due to the inherent symmetry of the model. The rigid body model does not, which is also expected due to the asymmetry between the model being perturbed one way versus the other. The data-driven model shows symmetry similar to that of the point-mass model; this is due to the assumption of linearity. Both models predict a symmetric shortening of the step for sideways perturbations, as shown in Figures 2b and d. That is, for a sideways velocity or position perturbation, both models move their next stance foot slightly backward, effectively taking a shorter step.

### 4. Discussion

Superficially, energy-optimal feedback control might seem implausible, as perhaps the energy it takes to reject small, incessant perturbations during normal walking is a small fraction of the normal walking energy cost. However, we find at least qualitative similarities between such optimal responses and human walking dynamics, as derived from intrinsic gait variability. In future work, we will compare such optimal responses back to steady-state with actual perturbation experiments, rather than just linear models derived from intrinsic gait variability. Further, we will examine optimal transients for higher DOF bipedal models so as to compare with both point-mass models and experimental data.

#### 5. Acknowledgments



Figure 2: Optimal and data-derived foot-placement responses to sideways position and velocity perturbations. The plots show the changes in the fore-aft and side-to-side position for the second step stance foot. A positive perturbation is away from the stance foot and a negative perturbation is towards the stance foot.

This work was supported by National Science Foundation grant 1254842.

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