

Non-linear robust control for inverted-pendulum 2D walking

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1 Introduction

We exemplify the design of a high-level controller for bipedal walking with a 2D point-mass inextensible-legs inverted-pendulum model. The key idea is that balance control authority is primarily from control of swing leg foot placement and magnitude of trailing-leg push-off. Our controller is defined implicitly as the solution of an optimization problem, described below.

The general form of the resulting optimized controller suggests a few simple principles for regulating walking speed: 1) The robot should take bigger steps both when speeding up and when slowing down 2) push-off is useful for regulating small changes in speed, but it is fully saturated or inactive for larger changes in speed. These results [1] build on the previous work of Zaytzev[2].

2 Walking Model

Our model: A point-mass body on mass-less in-extensible legs. The model has two control inputs at each step: the angle of the stance leg at heel-strike, and the push-off impulse that occurs immediately before heel-strike. We allow arbitrary placement of the swing leg restricted by reasonable bounds on step angle and push-off impulse.

Starting from a mid-stance where the stance leg is vertical and rotating clockwise, a step comprises four distinct phases: 1) The hip-mass swings down to the desired step angle; 2) an impulsive push-off impulse is applied on the trailing stance leg; 3) an impulsive heel-strike collision; and 4) the new stance leg rotates up to the next mid-stance.

3 Controller

The controller computes the optimal step angle and push-off using non-linear optimization: minimize the number of steps to converge to the target speed, while preventing falls, given a set of bounded disturbances. A separate informal loop in the optimization sets the bounded disturbances to be as large as possible while still yielding good solutions. The optimization can be computed in advance and stored for interpolated real-time use online.

We model disturbances as perturbations to the model (leg length), sensors (mid-stance speed), and actuation (push-off impulse and step angle). The controller is designed using a sub-set of disturbances (the 16 corners of the 4D disturbance cube). Robustness is then verified using random disturbances

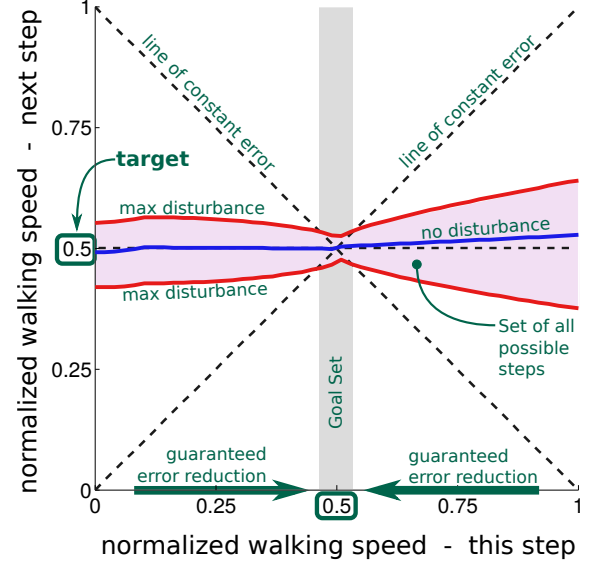


Figure 1: Stabilized step Map. Closed-loop walking dynamics, using our robust controller. With no disturbance the control is nearly deadbeat (blue horizontal line). With perturbations in model, sensors, and actuation, the controller prevents falls and keeps the motion in the pink region between the red lines.

sampled from the full disturbance set. We use Lyapunov functions to prove the stability and massive simulations (10^6 steps) as a check.

4 Discussion

This controller robustly stabilize slow, moderate, and fast walking gaits for the inverted pendulum model of walking, using parameters based on our robot, the Cornell Ranger. Missing from the discussion here is the less subtle problem of controlling the joint actuators to fulfill the high-level control commands from this controller. Our present intention is to use this controller on our new multi-DOF 3D robot.

References

- [1] M. Kelly and A. Ruina, “Non-linear robust control for inverted-pendulum 2D walking,” in *International Conference on Robotics and Automation*, (Seattle, WA), 2015.
- [2] P. Zaytzev, *The viability and controllability approach to robustness in bipedal locomotion: application to simple models*. PhD thesis, Cornell University, 2015.