How the arches of the feet influence stiffness

Shreyas Mandre[1], Marcelo Dias[2,3], Dhiraj Singh[4], Mahesh Bandi[4], and Madhusudhan Venkadesan[5]
[1] Brown University, Providence RI, USA
shreyas_mandre@brown.edu
[2] Aalto Science Institute, Aalto University, Espoo, Finland
[3] Nordic Institute for Theoretical Physics, Stockholm, Sweden
marcelo.dias@aalto.fi
[4] Okinawa Institute of Science and Technology, Okianawa, Japan
dhiraj.singh@oist.jp, bandi@oist.jp
[5] Yale University, New Haven CT, USA
m.venkadesan@yale.edu

1 Introduction

Motivated by a common observation that curved objects are stiffer than flat objects under bending (see Figure 1), we investigate the extent to which the arches of the human foot contribute to its stiffness for loading scenarios similar to observed when pushing off in walking and running. Mechanically, the foot is a force transmitting interface between the human body and the ground. The main mode of deformation we consider is one of longitudinal bending. We juxtapose this mechanism with another anatomical feature, the plantar fascia, that contributes to the stiffness. Our tools for comparison are mathematical analyses and experiments with analogue physical models that capture the essential ingredients of these features.

2 Shallow shell analysis

Our model for the foot consists of a "shallow shell", a thin elastic object of thickness t with a small curvature in its undeformed state. The radii of curvatures in the longitudinal direction (R_x) and the transverse direction (R_y), which mimic the two arches in the foot, are comparable or larger than L, the characteristic in-plane length of the object. The stiffness of this object is considered in comparison to that of an object without the curvatures (the special case of a flat plate). This stiffness is non-dimensionalized by the elastic properties of the plate Bw/L^3, where $B = Et^3/12(1-\nu^2)$ is the bending stiffness of the plate, w is its width, E is the elastic modulus of the material of the plate, and ν is its Poisson ratio.

The stiffness of these shells is computed numerically and measured experimentally. These shells are molded experimentally from poly-dimethyl siloxane using a 3D printed mold, clamped on one side, and loaded with controlled displacements, and we measure forces as the system's response. The ratio of the load to the displacement of the loaded side is the stiffness of the shell. The stiffness is also computed using a finite element simulation of the shell. The parameters varied are t, L, R_x, and R_y, and lead to a stiffness variation of several orders of magnitude.

The stiffness is plotted as a function of the curvatures in figure 2. For large radii of curvatures, the stiffness asymptotes to the stiffness of a flat plate. As the transverse radius of curvature is reduced, the stiffness starts to rise when $Rt/L^2 \sim O(1)$, and behaves like a power law for smaller transverse radii. The dependence on the longitudinal radius of curvature does not show such behavior. We find that the contribution from the transversal arch to the stiffness dominates over that from other features.

The role of the transverse arch is to couple the longitudinal bending mode to transverse in-plane stretching. Because stretching is much stiffer than bending, this coupling stiffens the foot. In the range $L^2/t \gg R_y \gg L$, in which the human foot may be considered to lie, the ratio of bending stiffness scales as $(R_y t/L^2)^{-3/2}$, which is an asymptotically large factor for a shallow shell. The absence of such a coupling between bending and stretching arising from the longitudinal curvature renders the contribution of the longitudinal arch much smaller than that of the transversal arch.

The plantar fascia are introduced in our analysis as an additional elastic material on one side of the shell. We find that the introduction of such material shifts the neutral plane of the deformation of the shell, but does not lead to any bending-stretching coupling. Therefore, the only mechanism for stiffening arises from the change in the bending stiffness. Furthermore, the elastic properties of the plantar fascia are comparable to the elastic tissue that provides the longitudinal bending stiffness for a "flat" foot. These circumstances leads us to conclude that the contribution to the stiffness from the additional material is comparable to the stiffness of the flat plate.

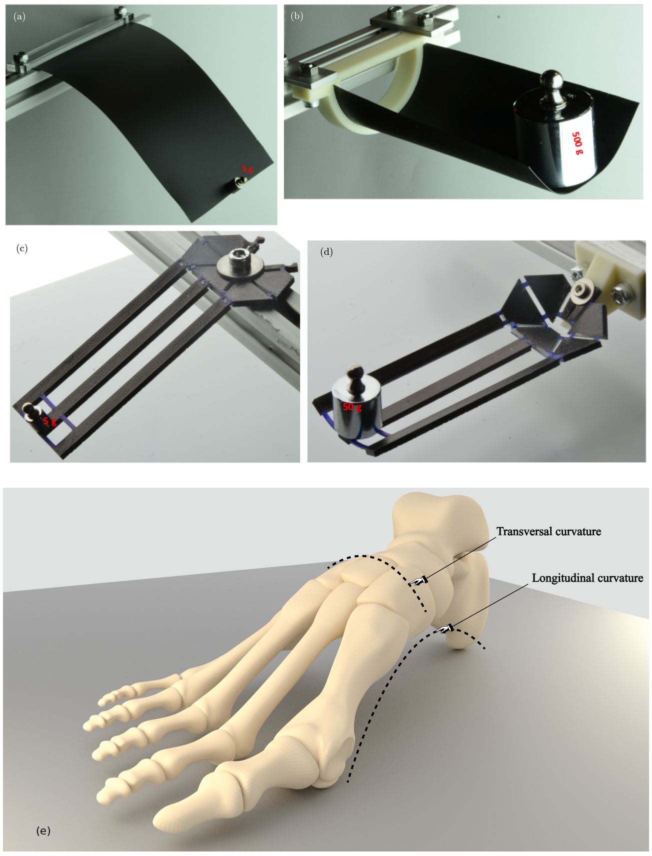

Figure 1: A simple experiment demonstrating the stiffening of continuum and discrete structures due to transverse curvature. (a) A sheet of card paper clamped at one end and supporting a 5 g weight. (b) Same sheet of card paper as in (a), but supported on one end by a curved clamp and subject to a 500 g weight on the other end. (c) A discrete structure 3D printed in a planar configuration to mimic the absence of a transversal arch deforms significantly under a load of 5 g. (d) The same discrete structure as in (c), but with a transversal arch can support a 50 g weight without significant deformation. (e) A rendition of the human foot skeleton showing the longitudinal and the transverse arch.

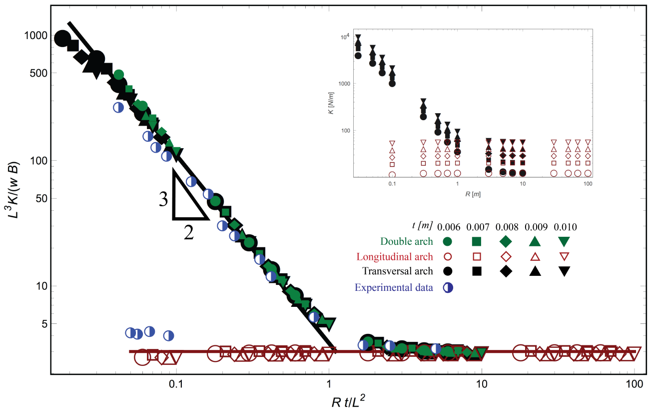

Figure 2: The stiffness of a shallow elastic as a function the transversal and longitudinal curvature. The stiffness K is non-dimensionalized by the stiffness of a flat plate, while the curvature R (either R_x or R_y) is scaled by a length L^2/t arising from the shallow shell analysis. The shells with a transverse curvature show an increase in the stiffness, which scales as $(Rt/L^2)^{-3/2}$, whereas the shells with only a longitudinal curvature show a stiffness comparable to that of the flat plate. Inset shows the stiffness data in dimensional form.

3 Conclusion

Repeated acceleration and deceleration of peripheral mass in walking and running has a direct increase in the metabolic energy consumption to cause such motion. Therefore, stiffness without the penalty of added mass is a desirable characteristic for an organ like the foot. The introduction of the transversal arch has a minimal increase in the mass of the foot, while increasing the stiffness. Based on our results, we argue that the transversal arch could be an evolutionary adaptation of the human foot, or a design feature in robotic or prosthetic feet, to increase its stiffness without substantially increasing the mass of the foot.