Contact Prediction in Optimal Motion Planning of Walking

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1 Introduction

The dynamics of biped systems during walking involves reaction forces and moments due to the unilateral constraint represented by the ground. Biped's trajectories and control inputs during walking are closely coupled with the contact force distribution in time and space as an indeterminate problem. For motion planning and control of such systems, the kinematic and kinetic redundancies must be resolved with optimization-based methods [1]. In the presence of unilateral constraints, the formulation of a nonlinear constrained optimization is required to concurrently solve for optimal motion, control inputs, contact status (i.e., when and where the feet make or break a contact with the ground), and contact response (i.e., ground reaction forces–GRF–and torques–GRT).

Fully predictive methods that concurrently solve for all these unknowns within a general optimization framework are still not available. This is a great challenge in the simulation, design, and control of general walking systems. Time stepping strategy for linear complementarity problem (LCP) [2], hybrid models [3], stochastic LCP [4], and dual variables transformation [5] have been introduced in the literature. However, most of them are based on a predefined sequence of contact time and position in order to find an optimal solution for both motion and contact response. A recent work considered this limitation by including the contact forces in the optimization algorithm as additional decision variables that satisfy nonlinear complementarity constraints [6]. Nevertheless, the need for *a priori* contact status information, time discretization strategies, integration schemes, and numerical difficulties are still common issues in the current approaches.

In this study, a novel approach for the optimal motion planning of multibody systems with contacts is established, based on sequential quadratic programming (SQP) algorithm for nonlinear programming (NLP). The objective is to predict and optimize the contact status and relative contact force within a unified optimization algorithm, while simultaneously optimizing the trajectories and satisfying all necessary physical, design, and task constraints.

2 Models

The biped model (Fig. 1) is a 4-link kinematic chain in the sagittal plane with 4 revolute joints and 4 point masses. The feet are not included in the chain. One complete step motion within the walking cycle is generated over the time interval [0, T], including single (SS) and double support (DS) phases. The SS/DS transition occurs at the contact time t_C between the left ankle P_4 and the ground. During the entire step motion, the right ankle P_0 is fixed to the ground (with no-slip condition), while P_4 clears the ground and covers a distance of $2sl$ meters in x direction.

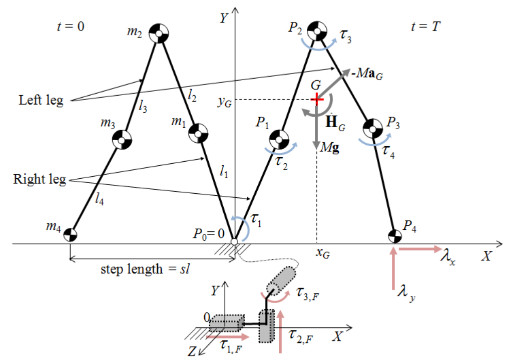

Figure 1: 3-global-DOF joint system and 4-DOF biped model at the initial and final instants of one step walking.

The foot-ground interaction is modeled through the resultant GRF acting at P_0 for right and P_4 for left leg. In this single-contact-point model, the right ankle joint (critical for stance) allows for non-zero right GRT, while no GRT are considered at the left ankle. The global coordinate system in sagittal plane is represented by 3 fictitious joints (2 translational, 1 rotational), with zero length and mass, connected to the biped through P_0. The complete generalized coordinate vector $\mathbf{q}(t) \in R^7$ includes fictitious joint variables $q_{j,F}$ and biped joint variables q_i, with j =1-3 and i =1-4. The corresponding generalized actuator torque vector $\boldsymbol{\tau}(t) \in R^7$ includes $\tau_{j,F}$ (the GRF and GRT at the right ankle) and τ_i (biped's joint torques, with $\tau_{3,F} = \tau_1$). The left GRF vector is $\boldsymbol{\lambda}(t) = [\lambda_x, \lambda_y]$. The system is governed by the Lagrange's equations of motion (EOM; [1]):

$$\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q},\dot{\mathbf{q}}) - \sum_i \mathbf{J}_i^T m_i \mathbf{g} - J_{\mathbf{q}}(\mathbf{q})^T \boldsymbol{\lambda} \qquad (1)$$

3 Formulation

Optimization Variables. The continuous joint trajectories $\mathbf{q}(t)$ are parametrized with the recursive 3rd degree B-spline [1], where 9 control vertices $x_{v,k}$ are selected for each joint variable (v =1,...,9, k =1,...,7). Actuator torques $\boldsymbol{\tau}(t)$, that includes right GRF and GRT, are calculated from EOM by inverse dynamics. Kinematic and dynamic functions and EOM are evaluated at each of the N total time steps. The GRFs λ_x and λ_y are also discretized for each time step h =1,...,N and calculated as optimization results, along with contact time t_c and position sl. The unknown optimization variables are $\{x_{v,k}, \lambda_{x,h}, \lambda_{y,h}, sl\}$.

Constraints. Kinematic and kinetic constraints related to the step task are implemented:

$$q_{1,F}(t) = q_{2,F}(t) = 0,\ P_4(T) = -P_4(0) = [sl,0],\ y_4(t) \geq \varepsilon(t) \quad (2)$$

$$\lambda_y(0) = \lambda_x(0) = \tau_{1,F}(T) = \tau_{2,F}(T) = \tau_{3,F}(T) = 0 \quad (3)$$

with ε as arbitrary ground clearance. Constraints at t =0, t =T for symmetric posture and velocity are imposed. The physics constraints related to contact dynamics are:

unilateral contact: $\tau_{2,F} \geq 0$, $\lambda_y \geq 0$, $\forall t$ (4)

friction cone: $\tau_{2,F}^2 \mu_S^2 - \tau_{1,F}^2 \geq 0$, $\lambda_y^2 \mu_S^2 - \lambda_x^2 \geq 0$, $\forall t$ (5)

sliding avoidance: $\lambda_y [x_4 - sl] = 0$, $\forall t$ (6)

closed form solution for normal left GRF (Fig.1), $\forall (t)$:

$$\lambda_y = \frac{1}{x_4}\left[\dot{H}_{G,z} + x_G M(\ddot{y}_G + g) - y_G M \ddot{x}_G - \tau_{3,F}\right] f_1(y_4) \quad (7)$$

closed form solution for tangential left GRF, $\forall (t)$, in case of small sliding ($\dot{x}_4 \neq 0$): $\dot{x}_4[\lambda_x - \mu_D \lambda_y f_2(\dot{x}_4)] = 0$ (8)

The static and dynamic friction coefficients are μ_S and μ_D. The term $f_1(y_4) = e^{-a\Delta y^2}$ is a correction that takes into account the complementarity nature of contact $\lambda_y \Delta y = 0$ (with $\Delta y = y_4 - y_0$ and y_0 = 0, Fig.2). For small sliding velocity, the term $f_2(\dot{x}_4) = -2[1/(1+e^{-2b\dot{x}_4}) - 1/2]$ serves to model a continuous and physically consistent friction force λ_x as a function of λ_y, μ_D, and $\dot{x}_4$ (Fig.2).

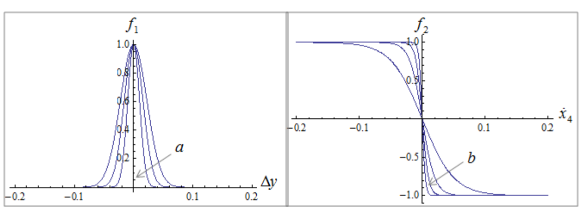

Figure 2: Functions $f_1(y_4)$ and $f_2(\dot{x}_4)$.

Cost Function. The function $f_0 = \int_0^T \sum_{i=1}^{4} \tau_i^2 dt \Big/ M^\alpha g^\beta sl^\gamma$ is minimized, which is similar to the walking cost of transport $COT = \int_0^T \sum_{i=1}^{4} |\tau_i \dot{q}_i| dt \Big/ Mgsl$ and is non-dimensionalized with $\alpha = 2$, $\beta = 3/2$, $\gamma = 5/2$. The COT is also evaluated for analysis purposes. Here f_0 is chosen for easier numerical implementation and gradient smoothness. However, none of the two measures represents the actual energy expenditure of the system (e.g., motors' electrical energy expenditure).

3 Results and Discussion

The above NLP is implemented in a C-based code including a custom multibody dynamics solver and the SNOPT SQP solver. For each simulation, the step frequency (1/T) and the lower admissible bound for sl (sl^{LB} = v^{MIN}/T) are given, with minimum velocity v^{MIN} of 0.5 m/s. The sl results (Tab. 1) are optimized from initial guesses (IG) and never reach sl^{LB}. The contact time t_c (not imposed *a priori*) is detected from GRF profile, for λ_y > 0.1 N. The trends of f_0 and COT agree. The approach naturally finds the optimal set of gait parameters for the given biped (gray, Tab. 1) and handles open and closed loop configurations (SS vs. DS) in a unified way. Left/right symmetry is found in optimal motion and contact response (Fig. 3).

Table 1: Optimization inputs and results.

Input values			
	(A) T = 0.4 s	(B) T = 0.5 s	(C) T = 0.6 s
sl^{IG} (m)	0.3	0.4	0.5
sl^{LB} (m)	67% sl^{IG}	63% sl^{IG}	60% sl^{IG}
Optimization results			
sl (m)	0.304 (101.3% sl^{IG})	0.355 (88.8% sl^{IG})	0.336 (67.3% sl^{IG})
t_c (s)	77.8% T	72.2% T	72.2% T
v (m/s)	0.76	0.71	0.56
f_0	1.45*10^{-1}	8.99*10^{-2}	7.53*10^{-2}
COT	0.338	0.246	0.198

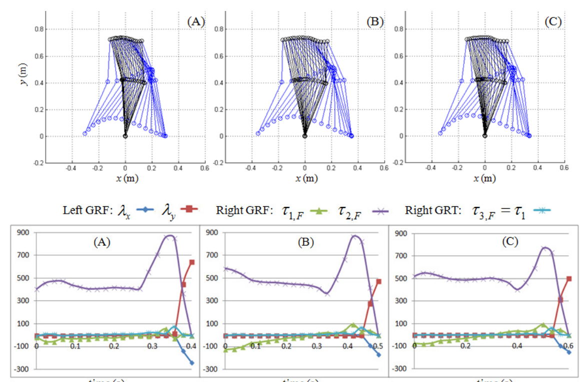

Figure 3: Optimized step motion (TOP) and contact response (BOTTOM) for (A)T=0.4s, (B)T=0.5s, (C)T=0.6s.

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