Subject-Specific Dynamic Model of Instantaneous Cost of Transport with Novel Heat Dissipation Formulation

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1 Introduction

Understanding metabolic energy expenditure (MEE) is critical in evaluating human movement [1]. The instantaneous cost of transport (ICOT) is a novel means of characterizing human gait on a time scale that was not previously possible. This is enabled by the joint-space model of MEE derived in this work that circumvents the limitations inherent in experimental measurements (e.g., phase delays, steady state and task restrictions, limited range of motion) or muscle-space models (e.g., complexities and indeterminacies, contacts and wrapping interactions, reliance on *in vitro* parameters). Although the first law of thermodynamics has been adopted in a few empirically-based inductive muscle energy models, the proposed model is based on comprehensive derivations as a deductive approach to account for all relevant work, heat, and energy components, and their rigorous formulations with respect to system boundaries, reference frames, relative/absolute quantities, and external/internal forces. The joint space consists of the generalized coordinates [2] of a combination of one or more kinematically equivalent revolute joints representing the resultant of multiple muscles that contribute to an anatomical joint movement. Relative to the muscle space, the joint space is finite dimensional with much fewer degrees of freedom (DOFs), yet uniquely determines whole-body segment configurations.

2 Model Derivation and Parameter Estimation

The joint angles representing the relative orientations between adjacent body segments serve as the generalized coordinates in joint space. If the whole body configuration is represented using n revolute joints (n DOFs), the joint space is composed of vectors of n generalized coordinates (q_i, $i = 1, \ldots, n$). The entire human body is identified as the system of interest in order to explicitly include the inherent metabolic energy level as one of the energy component terms in the first law of thermodynamics. The human body as the system of interest is constrained to be a closed system. When applied to the segment motion of the entire human body, the relevant energy components include the kinetic ($\dot{E}^k$), potential ($\dot{E}^p$; due to external conservative forces), internal potential ($\dot{U}^p$; strain due to internal conservative forces), and internal thermal ($\dot{U}^t$) energy rates, as well as the metabolic energy level. The first law can be written in terms of these relevant energy components, and a minus sign is attached to the MEE rate $\dot{E}^{met}$ to indicate the metabolic energy level:

$$\dot{E}^k + \dot{E}^p + \dot{U}^p + \dot{U}^t - \dot{E}^{met} = \dot{W}^{ext} + \dot{Q}^{ext} \tag{1}$$

where $\dot{W}^{ext}$ is the net work rate done by non-conservative external forces and moments that cross the system boundary and $\dot{Q}^{ext}$ is the net heat transfer rate across the system boundary. When combined with the work-energy principle of a multibody dynamic system, the above first law can be written for the MEE rate as the summation of internal work and heat, where the heat rate includes both the internal thermal energy and external heat dissipation rates. In this formulation, both the internal work and heat rates result from two sources: skeletal muscle activation and basal metabolism. The heat and work rate terms due to basal metabolism can be combined to constitute the basal metabolic rate (BMR). Then the total MEE rate can be written as:

$$\dot{E}^{met}(t) = \dot{W}^{mus}(t) + \dot{Q}^{mus}(t) + BMR \tag{2}$$

The instantaneous MEE rate model enables the evaluation of the dimensionless metabolic cost for each gait phase (COT) and time instant (ICOT), as follows:

$$ICOT(t) = \frac{\dot{E}^{met}(t)}{Mgv(t)} \tag{3}$$

where g is the gravitational acceleration, $v(t)$ represents the instantaneous walking speed, and T is the time duration. The COT can be evaluated for any time interval of interest [0, T], not only for an entire gait cycle, but also for its breakdown into gait phases.

A new formulation for heat rate is derived in this study. The heat rate of each i^{th} DOF is modeled as a function of kinematic variables ($q, \dot{q}$), dynamic variables ($\tau, \dot{\tau}$) and a set of k system parameters ($\alpha_1, \ldots, \alpha_k$). The system parameters can be subject-specific or DOF-specific. The two angular variables uniquely describe the trajectories of the whole body and by analogy the two torque variables uniquely describe the forces acting on the whole body system as well. The inclusion of the time-derivative of torque $\dot{\tau}$ as a dynamic variable allows the dissipation rate to vary with different loading rates: $\dot{Q}_i^{mus} = \dot{Q}_i^{mus}(q, \dot{q}, \tau, \dot{\tau}, \alpha_1, \ldots, \alpha_k)$. The dissipation function must satisfy two conditions. It cannot be negative because muscles cannot reabsorb heat to perform useful work, so therefore $\dot{Q}^{mus}(t) \geq 0 \quad \forall t$. The second condition is that the dissipation rate should vary with joint torque when the joint is stationary because holding a static position at a higher joint torque requires increased physical effort. This condition is implemented as follows:

$$\dot{Q}_i^{mus}(\tau_1) \neq \dot{Q}_i^{mus}(\tau_2) \text{ if } \tau_1 \neq \tau_2 \text{ and } \dot{q} = \dot{\tau} = 0 \qquad (4)$$

The form of the heat dissipation function is estimated using a dimensional analysis approach. Initially, only the kinematic and dynamic variables are considered in dimensional analysis. Four variables, $q, \dot{q}, \tau$, and $\dot{\tau}$, are grouped to form non-dimensional variables, such as $\tau\dot{q}/\dot{\tau}$. Relations between the non-dimensional variables and the heat dissipation rate are obtained from applying statistical methods to the experimental MEE data. The experimentation with various combinations yield relationships between joint variables and heat dissipation. The dimensional analysis is used to generate a general formulation independent of the sign of the work done. The dimensional analysis can be refined by considering additional relevant DOF-specific or subject-specific system parameters in the non-dimensionalization process. Any resulting form for the dissipation function must satisfy the two conditions of being non-negative and changing with torque even in a stationary position. After the form of the dissipation function has been established, it can be incorporated into an MEE model.

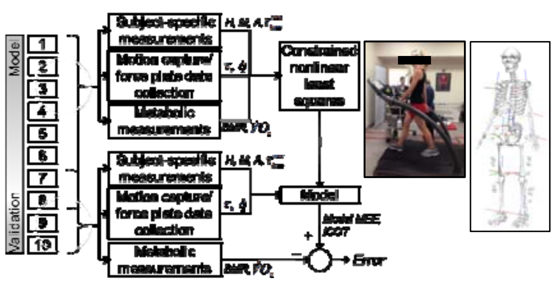

Figure 1: Data processing flow chart (left), metabolic testing (inset), and full body marker sets (right)

Ten healthy subjects participated in this study that included metabolic testing, kinematic and kinetic testing (Fig. 1), and strength testing. The protocol consisted of walking (on a treadmill for indirect calorimetry then over ground for kinematic and kinetic data collection) with rest periods of two minutes between five different walking speeds: 70%, 85%, 100%, 115%, and 130% of the preferred walking speed. This was followed by peak knee torque assessment on a dynamometer. Measured MEE rate, kinematic and kinetic variables, and subject-specific τ^{max} of the subjects were inputs to the model, and the heat coefficient functions were estimated using a constrained least squares formulation. The resulting heat coefficients were applied to the model to estimate MEE and ICOT for the separate validation subjects.

3 Results and Discussion

The mean of the time-averaged model MEE rates for the validation group are within 5.7 ± 4.6% absolute error of the experimental values. A strong inter- (R^2 = 0.98) and intra-subject (0.90 - 0.98) correlations were noted (Fig. 2).

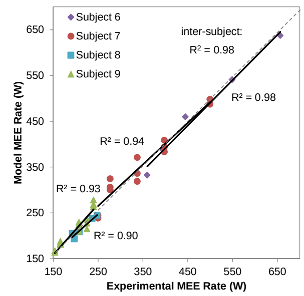

Figure 2: Correlation between average model MEE rates and experimental MEE rates for all validation trials.

The total COT and ICOT model results show the instantaneous metabolic cost as a function of percent gait cycle for 70%, 100%, and 130% of the preferred walking speed (Fig. 3). While the total COT can be obtained from both the experiments and the model, the ICOT are available only through the proposed model. The peaks in ICOT follow a pattern such that the faster the walking speeds are, the earlier in the gait cycle the peaks are detected. The ranges between the maximum and minimum ICOT values are larger at faster walking speeds.

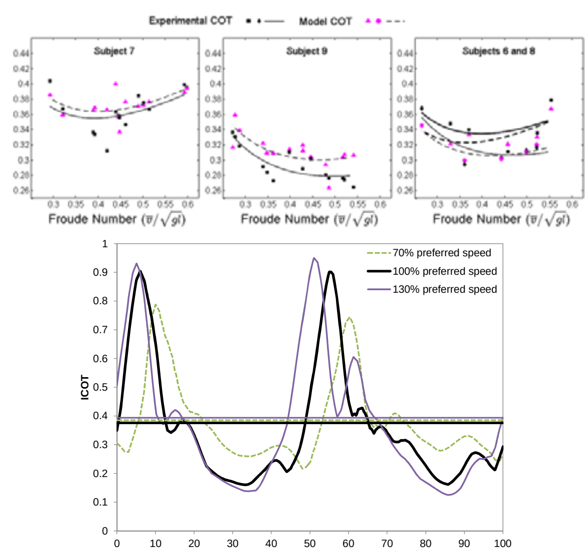

Figure 3: Model and Experimental COT (top) and model ICOT (bottom) for all validation data

References

[1] Long and Srinivasan, *J. R. Soc. Interface*, 2013.
[2] Kim et al., *Multibody Syst. Dyn.*, 2010.