Uncertain Multiple Contacts: A New Class of Bio-Inspired Controllers

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1 Overview

Many of the models used in the study of locomotion and manipulation are "hybrid – the equations of motion governing them undergo a discontinuous change at surfaces ("guards") corresponding to changes in the contact state. Furthermore, even when driven by deterministic internal commands the transition sequences themselves – the order of footfalls or finger contacts – are often uncertain due to external influences. We have developed new methods for integration and analysis of this class of hybrid dynamical systems, allowing us to compute the stability properties associated with trajectories passing through intersections of guards. These suggest potential advantages associated with multiple contact gaits such as quadruped trots and paces, and hexapod alternating tripod gaits. Our theoretical results allowed us to design and prove the correctness of a piecewise constant state-feedback law that produced a stable alternating tripod gait for the RHex robot. The controller is "nearly" distributed – it requires only one message request-response exchange between the legs per stride.

2 Motivation / State of the Art

One of the quintessential properties of legged locomotion is the change in contact states. Legs engage with the substrate for a time, then lift off and engage with it again at another location. When engaged, the legs carry the load of the body, resisting gravity and maintaining motion in the desired direction. When a leg transitions from "swing" to "stance" or back, the equations of motion governing the body undergo a rapid change in constraints that is often modeled by a hybrid transition through a guard (a discontinuity along a surface) representing the contact creation or destruction condition.

Many gaits posses a symmetry that implies that multiple legs touch down simultaneously[3;4]. Of course, in practice contacts are established nearly simultaneously, and the order with which they are established varies from cycle to cycle. Examples include multiple-contact gaits of polypedal animals, like trotting or pacing in tetrapod vertebrates, alternating tripod gaits in insects, and alternating tetrapod gaits in decapods like spiders and crabs. In running humans[1], feet can "heel-strike" or "toe-strike", and a given individual may alternate among these in a single run or even land "flat".

If the rhythmic motions associated with locomotion in these cases are viewed as a periodic solution of an underlying hybrid dynamical system, this periodic solution runs close to, or through, the intersection of multiple guards representing the variety of contacts that can be established. Local stability of such solutions cannot be ascertained with classical tools of dynamical systems because the flow in that region is not differentiable, even in those cases where classical results (perhaps relating to internal gait regulation) can prove its existence.

3 Our Approach

We have recently proven some core theorems showing the existence and uniqueness of flows in a large class of hybrid dynamical systems ("Event Selected r-Smooth"[2]) in which typical locomotion models fall. For those systems we also showed generalized ("Bouligand"[5]) differentiability with respect to initial conditions, and structural stability of the flow with respect to a Sobolev-like norm.

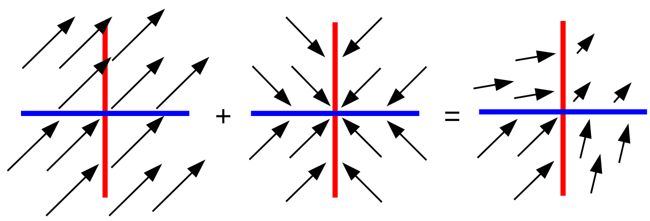

Figure 1: A stabilizing ("contractive") piecewise constant flow can be seen as a hybrid sink superposed on an ambient uniform flow

Our results become easier to interpret if the vector fields of the dynamical system, which are smooth in each domain, are replaced with a constant, "averaged" vector. Similarly, the arbitrary smooth manifold of each guard is replaced by a hyperplane. The resulting system can be viewed as a superposition of an ambient uniform flow and a uniquely hybrid, piecewise constant sink (Figure 3).

With such a leading order approximation of the dynamics we can demonstrate the core results and provide the computational formulae needed to assess stability. The same intuition can then be extended to the general piecewise smooth case.

4 Robot Controller Results

To explore the empirical relevance of our theoretical results, we constructed a piecewise constant vector field as a state-feedback control law for the hexpedal RHex robot. The robot configuration was modeled as a 6-torus of leg shaft angles. Several flat ("affine") guards were placed on the phase space associated with this 6-torus, and a constant vector field was asserted in each volume of the partition thus created. These piecewise constant vector fields can be shown, using our tools, to synchronize the legs within each tripod, and to run the tripods through fast swing and slow stance motions in an alternating fashion. The robot programmed with such a controller reliably executes an alternating tripod gait, even when encountering unexpected objects on the ground – yet does so without any "reference trajectory".

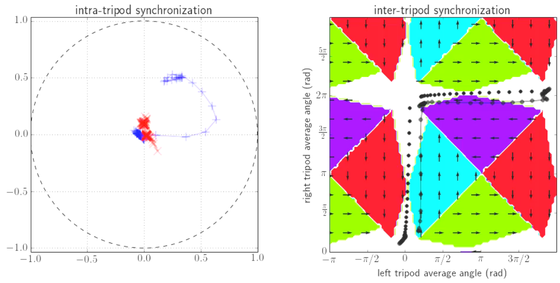

Figure 2: Visualization of the intra-tripod synchronization (left; tripods in blue and red), by projecting the 3-torus of leg phases along its diagonal. Inter-tripod synchronization visualized as a map of the 2-torus of tripod average angles (right; black dots robot state), with the piecewise constant control law (colored polygons and arrows).

Figure 3: Robot walking with our controller

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