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Biomechanics

The metabolic cost of changing walking speeds is significant, implies lower optimal speeds for shorter distances, and increases daily energy estimates

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Humans do not generally walk at constant speed, except perhaps on a treadmill. Normal walking involves starting, stopping and changing speeds, in addition to roughly steady locomotion. Here, we measure the metabolic energy cost of walking when changing speed. Subjects (healthy adults) walked with oscillating speeds on a constant-speed treadmill, alternating between walking slower and faster than the treadmill belt, moving back and forth in the laboratory frame. The metabolic rate for oscillating-speed walking was significantly higher than that for constant-speed walking (6–20% cost increase for ± 0.13–0.27 m s^{-1} speed fluctuations). The metabolic rate increase was correlated with two models: a model based on kinetic energy fluctuations and an inverted pendulum walking model, optimized for oscillating-speed constraints. The cost of changing speeds may have behavioural implications: we predicted that the energy-optimal walking speed is lower for shorter distances. We measured preferred human walking speeds for different walking distances and found people preferred lower walking speeds for shorter distances as predicted. Further, analysing published daily walking-bout distributions, we estimate that the cost of changing speeds is 4–8% of daily walking energy budget.

1. Introduction

Walking in typical human life requires changing speeds. Most daily walking appears to happen in short bouts [1], starting and ending at rest. Here, to better understand such behaviour, we measure the metabolic cost of changing walking speeds. Although much is known about constant-speed walking [2,3], the cost of changing speeds has not been measured without non-inertial treadmill speed changes or step-frequency control [4]. Here, we show that the cost of changing speed is significant and an appreciable fraction of daily walking energy budget. This cost may have behavioural implications: we predict lower optimal walking speeds for short distances; we then measured and found that our subjects prefer lower speeds for shorter distances.

2. Material and methods

(a) Experiment: metabolic cost of oscillating-speed walking

Subjects ($N = 16$, 12 males and 4 females, 23.25 ± 2.1 years, height 177.08 ± 7.4 cm, mass 75.99 ± 12.94 kg, mean $\pm$ s.d.) performed both 'steady' (constant-speed) and 'oscillating-speed' walking trials. Oscillating walking speeds were achieved on a constant-speed treadmill by alternately walking faster and slower than the belt (figure 1a). Two distinct audible tones of durations T_{fwd} and T_{bck} alternated in a loop indicating whether the subjects should move towards the treadmill front or rear. We used three ($T_{\mathrm{fwd}}, T_{\mathrm{bck}}$) combinations, (1.9,1.9) s, (2.8,2.8) s and (1.9,2.8) s, obtaining different
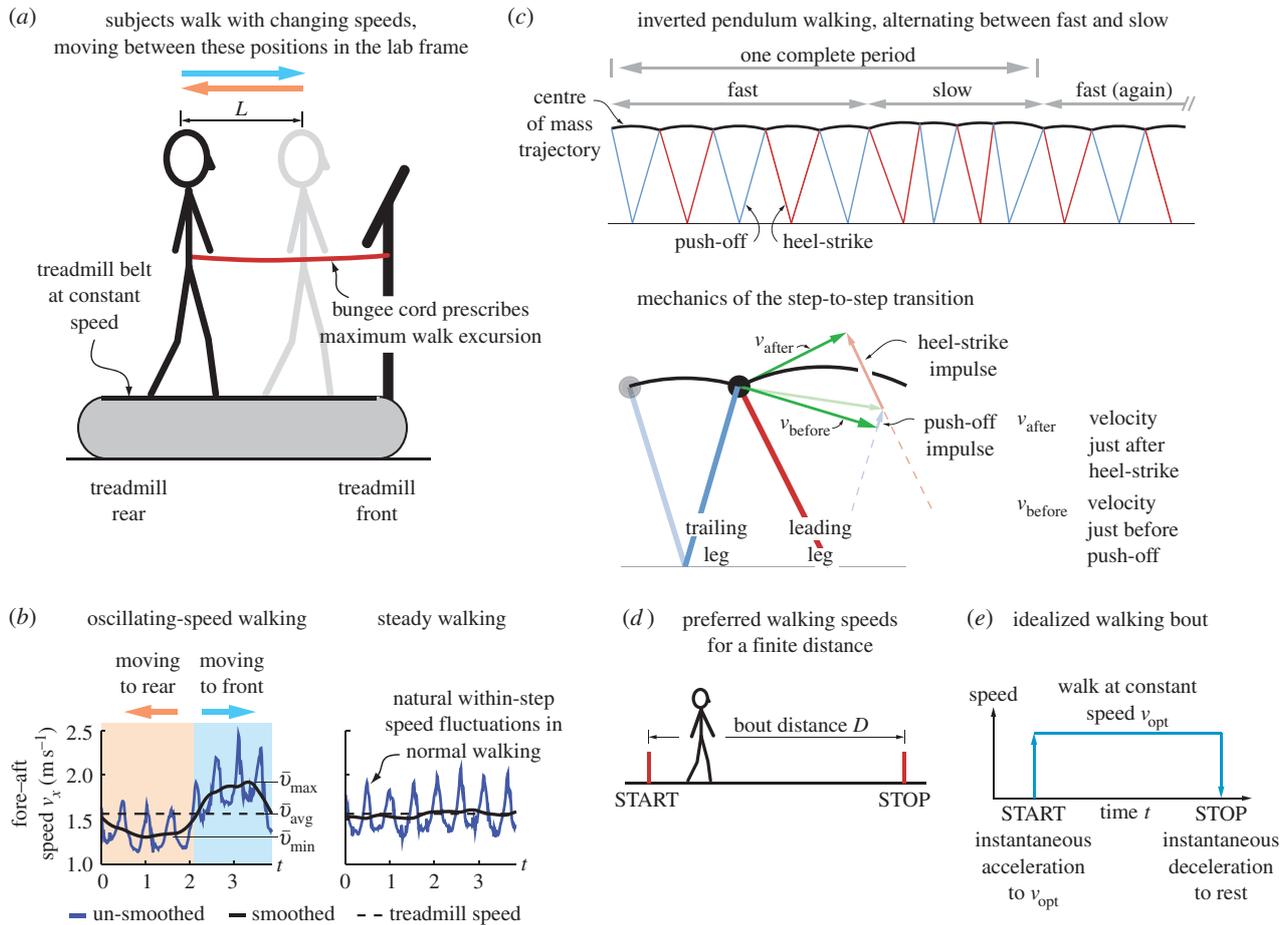
Figure 1. Experimental protocols and theoretical models. (*a*) Subject walking with oscillating speeds on a constant-speed treadmill, walking faster and slower than the belt, moving between two prescribed positions. A longitudinal bungee cord (never to be made taut) constrains the rear-most position and a bungee cord perpendicular to sagittal plane (not shown, never to be touched) constrains the forward-most position. (*b*) Sacral marker fore–aft velocities, original and smoothed. (*c*) A nine-step periodic inverted pendulum walking motion, with five steps faster and four steps slower than the mean speed. Initial and final stance-leg directions are shown for each step (red and blue). Details of one step-to-step transition are shown; downward velocity at the end of one step is redirected by push-off and heel-strike impulses. (*d*) Measuring preferred walking speeds as a function of bout distance D; subjects start and stop at rest. (*e*) An idealized bout: human travels the whole distance D at single speed v_{opt}, starting and stopping instantaneously.

speed fluctuations. We instructed subjects to walk between fixed positions on the treadmill (0.48 m apart) giving mean excursion length $L = 0.41 \pm 0.08$ m (figure 1*a*). The subjects obeyed the imposed back-and-forth time period constraints: mean periods differed from prescribed periods by $0.97 \pm 0.24\%$. While humans do not usually walk with oscillating speeds, this protocol was designed to isolate the cost of changing speed.

Oscillating-speed trials were at one or both constant treadmill speeds 1.12 and 1.56 m s^{-1} (equal to the mean speeds): 10 subjects at both speeds, four subjects at 1.12 m s^{-1} only and two at 1.56 m s^{-1} only, with random speed order. Steady walking trials were performed at speeds ranging from 0.89 to 1.78 m s^{-1}, including 1.12 and 1.56 m s^{-1}.

Metabolic rate per unit mass (W kg^{-1}) was estimated using respirometry (Oxycon Mobile), approximated as $\dot{E} = 16.58 \dot{V}_{O_2} + 4.51 \dot{V}_{CO_2}$ ($\dot{V}$ in ml kg^{-1} s^{-1}), denoted $\dot{E}_{steady}$ and $\dot{E}_{osc}$ for steady- and oscillating-speed trials, respectively. Trials lasted 7 min: 4 min to reach metabolic steady state and 3 min to estimate the mean metabolic rate. The speed oscillation periods (3.8–5.6 s) are much smaller than typical metabolic time-constants (30 s), so our metabolic steady state is nominally constant. A sacral marker's motion was measured with marker-based motion capture.

(b) Experiment: preferred walking speed

Subjects ($N = 10$) were asked to walk ten distances ($D = 0.5, 1, 2, 4, 6, 8, 10, 12, 14$ and 89 m) at a comfortable speed, starting and

ending at rest (figure 1*d*). We had three trials per distance, all trials in random order, but performed 0.5–1 m trials separately.

(c) Model 1: kinetic energy fluctuations

In this model, we attribute the metabolic cost increase for oscillating-speed walking over steady walking to fore–aft kinetic energy fluctuations beyond what happens within each step in constant-speed walking. Figure 1*b* shows fore–aft velocity $v_x(t)$ of the sacral marker for oscillating-speed and steady walking, approximating centre of mass motion. Smoothing $v_x(t)$ with an averaging window equal to step period gives $\bar{v}_x(t)$, removing within-step speed fluctuations (figure 1*b*). The mass-normalized metabolic cost increase for oscillating-speed walking over steady walking due to the kinetic energy fluctuations for each cycle is modelled as $\Delta E_{ke} = (\bar{v}_{max}^2 - \bar{v}_{min}^2)(\eta_{pos}^{-1} + \eta_{neg}^{-1})/2$, where $\bar{v}_{max}$ and $\bar{v}_{min}$ are maximum and minimum smoothed fore–aft speeds for that cycle and $\eta_{pos} = 0.25$ and $\eta_{neg} = 1.2$ are typical positive and negative muscle work efficiencies [5]. The model-predicted metabolic rate increase $\Delta \dot{E}_{ke}$ for each oscillating-speed trial was the median $\Delta E_{ke}/T_{period}$ over all cycles.

(d) Model 2: inverted pendulum walking

We consider inverted pendulum walking of a point-mass biped, for which the total walking metabolic cost is the sum of (i) a step-to-step transition cost (described below) and (ii) a leg-swing cost [6]. Using
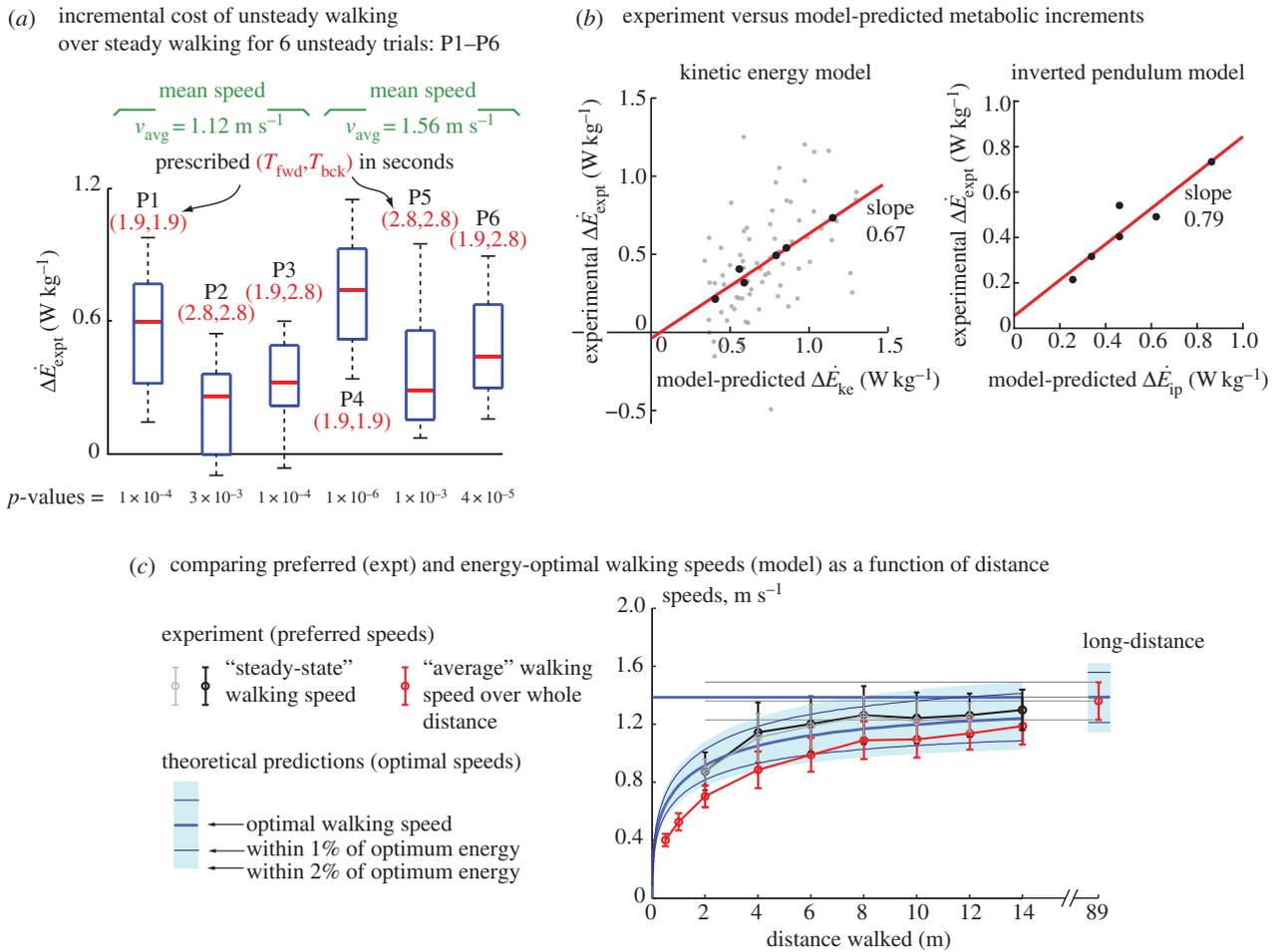
Figure 2. (a) Difference $\Delta\dot{E}_{expt}$ between oscillating- and constant-speed walking metabolic rates for six oscillating-speed trials (P1–P6): three (T_{fwd}, T_{bck}) combinations and two mean speeds. Box plot shows median (red bar), 25–75th percentile (box), and 10–90th percentile (whiskers); p-values use one-sided t-tests for the alternative hypothesis that metabolic rate differences are from a distribution with greater-than-zero mean. (b) $\Delta\dot{E}_{expt}$ compared with kinetic energy model $\Delta\dot{E}_{ke}$ and inverted pendulum model $\Delta\dot{E}_{ip}$; we show experimental and model means (black filled circles), the best-fit line (red, solid) and all subjects' trials (scatter plot, grey dots); no scatter plot for inverted pendulum model as it produces only one prediction per trial. (c) Distance-dependence of the model-based energy-optimal walking speeds (blue, solid) and experimentally measured preferred speeds (red and black error bars). Ranges of model-based energy-optimal speeds within 1% (blue line, thin) and 2% (blue band) of optimal energy cost are shown. We show whole-bout 'average' speeds (red) and 'steady-state' speeds over middle 1.42 m (black, thick), indistinguishable from over middle 0.75 m (grey, thin). Average preferred speeds for 0.5–14 m trials were significantly lower than that for the 89 m trial (paired t-test, $p < 0.01$); similarly, the 'steady-state' speeds for 2–6 m were significantly lower than that for 89 m (paired t-test, $p < 0.04$).

numerical optimization, we found the multi-step-periodic inverted pendulum walking motion (e.g. figure 1c) satisfying our oscillating-speed experimental constraints and minimizing this metabolic cost. Here, we derived and used expressions for step-to-step transition cost for non-constant-speed walking, generalizing previous constant-speed expressions (see electronic supplementary information). This step-to-step cost accounts for the push-off and heel-strike work to redirect the centre of mass velocity during the step-to-step transition (figure 1c), and depends on leg-angles and centre of mass velocities. The model prediction $\Delta\dot{E}_{ip}$ is the difference between the optimal oscillating-speed and constant-speed costs at the same mean speed. A metabolic cost term proportional to the integral of leg forces contributed almost equally to oscillating-speed and constant-speed walking costs and did not contribute to their difference (see the electronic supplementary material for details).

3. Results

(a) Metabolic rate of oscillating-speed walking

Metabolic rates for all six oscillating-speed trials (P1–P6, figure 2a) were significantly higher than the corresponding steady-state costs. Metabolic rate increment over constant-

speed walking $\Delta\dot{E}_{expt}$ was significantly greater than zero for all trials (one-sample t-test, for all $p < 2 \times 10^{-3}$, figure 2a). Oscillating-speed trials with higher speed fluctuations had higher metabolic rates with one exception (P1 > P3 > P2, P4 > P6 and P4 > P5, all $p < 0.02$).

(b) Model predictions

Both the kinetic energy fluctuation model $\Delta\dot{E}_{ke}$ and the inverted pendulum model $\Delta\dot{E}_{ip}$ were correlated with measured metabolic rate increments $\Delta\dot{E}_{expt}$ (figure 2b). The kinetic energy model and experimental costs are best-fitted by the line: $\Delta\dot{E}_{expt} = \lambda_{ke}(\Delta\dot{E}_{ke}) - 0.04$, with $\lambda_{ke} = 0.67$ whether we use trial means ($R^2 = 0.96$, 95% CI of $\lambda_{ke} = 0.48$–0.86) or all data ($R^2 = 0.24$, 95% CI of $\lambda_{ke} = 0.39$–0.95). Similarly, the inverted pendulum model and experimental costs are best-fitted by $\Delta\dot{E}_{expt} = \lambda_{ip}(\Delta\dot{E}_{ip}) + 0.05$ with $\lambda_{ip} = 0.79$ ($R^2 = 0.88$, 95% CI of $\lambda_{ip} = 0.39$–1.19).

(c) Daily energy budget for starting and stopping

Humans mostly walk in short bouts [1]. For simplicity, we idealize a bout of distance D and mean speed v as

instantaneously accelerating from rest to speed v, walking at constant speed v and stopping instantaneously at time D/v (figure 1e). The total metabolic energy per unit mass $E_{bout}(D,v)$ for this idealized bout has two components: (i) a starting and stopping cost, extrapolating from the kinetic energy model, $\lambda_{ke}(\eta_{pos}^{-1} + \eta_{neg}^{-1})v^2/2$ and (ii) a cost for steady-state walking at speed v, given by $\dot{E}_{steady} = a + bv^2$ with $a = 2.22$ W kg^{-1} and $b = 1.15$ W kg^{-1} (m s^{-1})$^{-2}$ [2], so that $E_{bout} = \lambda_{ke}(\eta_{pos}^{-1} + \eta_{neg}^{-1})v^2/2 + (a + bv^2)D/v$. Applying this model to data in [1] with $v_{opt} = \sqrt{a/b} = 1.39$ m s^{-1} and the 95% CI of λ_{ke} suggests that starting–stopping costs are 4–8% of daily walking energy expenditure (electronic supplementary material); this cost fraction (4–8%) may apply primarily to the subject population of [1], adults working in offices, but could be estimated for other populations given the distribution of their daily walking bout lengths.

(d) Optimal and preferred walking speeds are lower for shorter distances

For the idealized bout of distance D (figure 1e), the energy-optimal walking speed v_{opt} that minimizes $E_{bout}(D,v)$ is given by the implicit function: $\lambda_{ke}v_{opt}^3(\eta_{neg}^{-1} + \eta_{pos}^{-1})/(a - bv_{opt}^2) = D$. This metabolically optimal speed increases with distance D, approaching $v_{opt} = \sqrt{a/b}$ for large distances (figure 2c).

As predicted by the distance-dependence of optimal walking speeds, preferred human walking speeds in our experiment, both 'average' and 'steady-state' speeds, increased with distance (figure 2c). 'Average' preferred speed is the mean speed over the whole bout; a proxy for the 'steady-state' preferred speed is the mean over the bout's middle 0.75 m (indistinguishable from averaging the middle 1.4 m). Model-predicted optimal speeds have a 0.96 correlation coefficient (Pearson's) with experimental steady-state preferred speeds, which were within 1–2% optimal cost. Our subjects could accelerate to higher mean or steady-state speeds, but they preferred not to. Therefore, the time taken to accelerate–decelerate cannot explain lower speeds for shorter distances.

4. Discussion

We have shown that oscillating-speed walking costs more than constant-speed walking. These cost-increments are correlated with kinetic energy fluctuation and inverted pendulum model predictions; inverted pendulum model predictions were closer to experimental values (regression slope closer to 1), perhaps because the kinetic energy model ignores walking mechanics. The cost of changing speeds implies lower energy-optimal speeds for shorter distances, reflected in our preferred speed experiments here and previous amputee data [7].

Preferred walking speeds are used to quantify mobility and rehabilitation [8], so bout distances should be chosen to avoid artificially lowering speeds. Using the cost of changing speeds may improve daily activity tracking, energy balance estimations for obesity, and metabolic estimations during sports (e.g. soccer [9]).

A previous experiment [4] considered walking with greater speed fluctuations (± 0.15 to ± 0.56 m s^{-1}) than our study (± 0.13 to ± 0.27 m s^{-1}) and similar kinetic energy fluctuations (electronic supplementary material), and found significant cost increase over steady walking for their highest speed fluctuations. However, this study [4] required walking on an oscillating-speed treadmill belt or controlling step durations in overground walking (derived from oscillating-speed treadmill trials). An oscillating-speed treadmill, being a non-inertial frame (in contrast to a constant-speed treadmill), can perform mechanical work, and is not mechanically equivalent to overground oscillating-speed walking (as noted in [4]). Further, prescribing step durations to control overground speed fluctuations is different from prescribing speed fluctuations directly [10].

Future work could involve overground experiments (say by having subjects follow a laser projection [11]), detailed biped and metabolic cost models (including muscle force and history dependence), using different speed fluctuations and measuring metabolic cost while subjects alternate between walking, stopping and starting (being directly applicable to walking bouts, relying less on extrapolation).

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Supplementary Information Appendix for
The metabolic cost of changing walking speeds is significant, implies lower optimal speeds for shorter distances, and increases daily energy estimates

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S1 Deriving step-to-step transition work for changing speeds

The mechanical work done to redirect the velocity of the center of mass from downward to upward when transitioning from one step to the next is thought to be a major determinant of the metabolic cost of walking at a constant speed [1, 2, 3].

Mathematical expressions for this step-to-step transition cost have previously been derived for such steady state walking, with the simplifying assumption that humans walk with an inverted pendulum gait [4, 1, 2, 3]. When a person walks at continuously varying speeds, as in our experiments, the step-to-step transition will include a change in both the magnitude and direction of the COM velocity. Here, we derive an expression for the work done in the step-to-step transition when walking at changing (non-constant) speeds, thereby generalizing previous work [4, 1, 2, 3]. We allow that the length of the leg to change during the step-to-step transition (unequal θ_{before} and θ_{after}) while having constant leg length during the stance phase.

Push-off before heel-strike. Figure S1 describes the transition from one inverted pendulum to the next using push-off and heel-strike impulses; panels a-c describe situations in which the push-off happens entirely before heel-strike, which we consider first. In particular, Figure S1a-b shows a finite reduction in speed being accomplished during the step-to-step transition, with push-off before heel-strike. We focus on Figure S1b in the following derivation. Here, vector $\overrightarrow{\text{OA}}$ with magnitude $\text{OA} = V_{\text{before}}$ and making angle θ_{before} with horizontal, is the body velocity just before push-off at the end of one inverted pendulum phase. Vector $\overrightarrow{\text{OC}}$, with magnitude $\text{OC} = V_{\text{after}}$ and making angle θ_{after}, is the body velocity just after heel-strike at the beginning of the next inverted pendulum phase.

A push-off impulse is applied along the trailing leg to change velocity $\overrightarrow{\text{OA}}$ to $\overrightarrow{\text{OB}}$, along $\overrightarrow{\text{AB}}$. Then, a heel-strike impulse is applied along the leading leg to change velocity $\overrightarrow{\text{OB}}$ to $\overrightarrow{\text{OC}}$, along $\overrightarrow{\text{BC}}$. The push-off positive work W_{pos} is the kinetic energy change from $\overrightarrow{\text{OA}}$ and $\overrightarrow{\text{OB}}$ given by $\frac{1}{2}m\text{OB}^2 - \frac{1}{2}m\text{OA}^2$ and the heel-strike negative work is the kinetic energy change from $\overrightarrow{\text{OB}}$ to $\overrightarrow{\text{OC}}$ given by W_{neg} given by $\frac{1}{2}m\text{OC}^2 - \frac{1}{2}m\text{OB}^2$, which simplify to:

$$W_{\text{pos}} = \frac{1}{2}m\,(\text{AB})^2 \quad \text{and} \quad W_{\text{neg}} = \frac{1}{2}m\,(\text{BC})^2 \quad \text{respectively.} \tag{1}$$

As opposed to the steady walking situation [4, 1, 2, 3], when changing speeds, the step-to-step push-off positive work W_{pos} and the step-to-step heel-strike negative work W_{neg} will be unequal.

First, we note that angle that in triangle EBD, $\angle\text{DEB} = \pi/2 - \theta_{\text{after}}$, $\angle\text{BDE} = \pi/2 - \theta_{\text{before}}$, and $\angle\text{EBD} = \theta_{\text{after}} + \theta_{\text{before}}$. We use the geometric relations that $\text{AB} = \text{AD-BD}$, $\text{BC} = \text{EB+CE}$, $\text{AD} = \text{OA}\tan\theta_{\text{before}}$, $\text{CE} = $
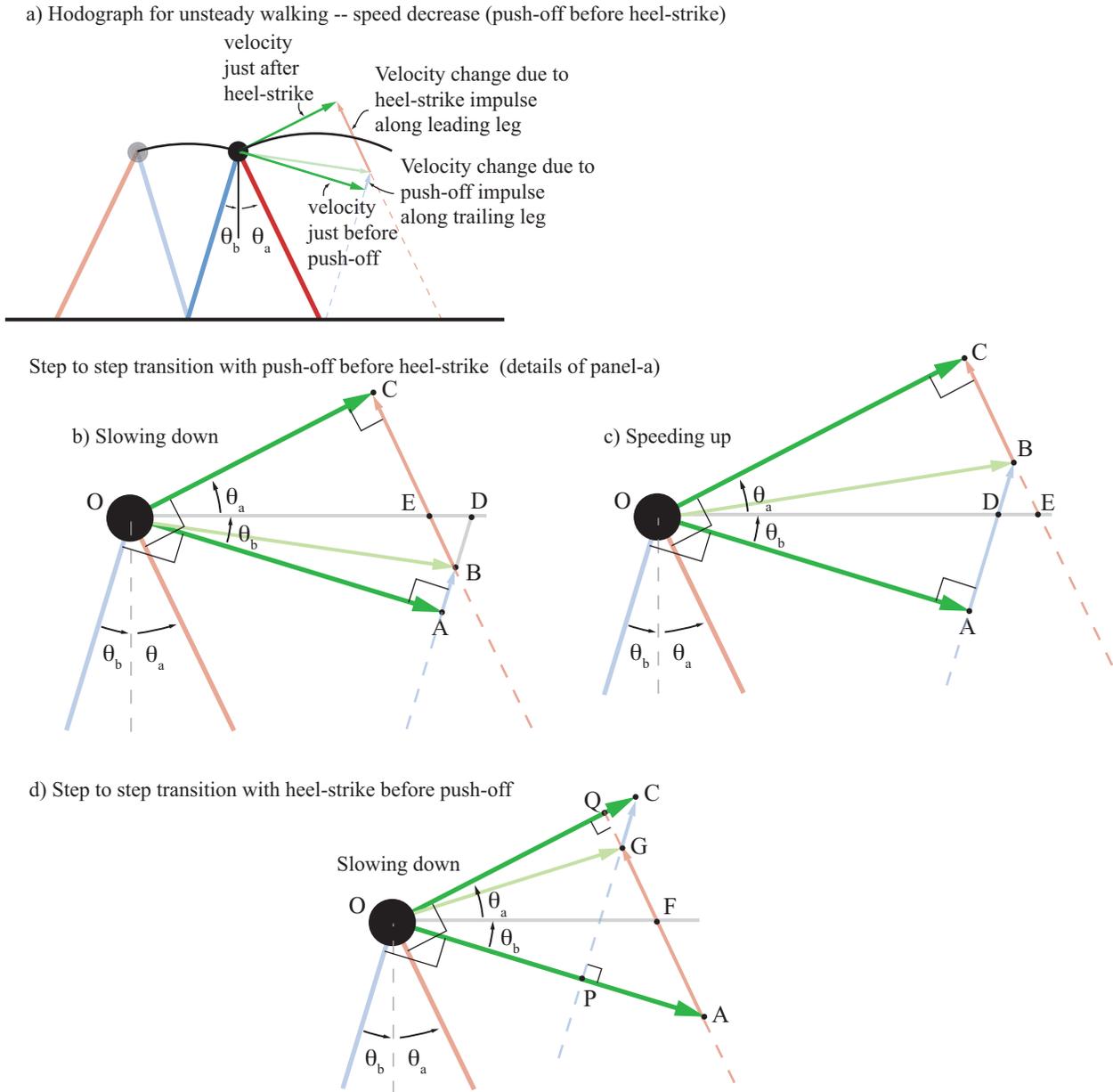
Figure S1: Step-to-step transition to change speed. a) The walking motion is assumed to be inverted pendulum-like with the transitions from one inverted pendulum step to the next accomplished using push-off and heel-strike impulses. Overlaid is the 'hodograph' (a depiction of velocity changes) during the step-to-step transition, when push-off happens entirely before heel-strike. b) Details of the velocity changes during step-to-step transition, with push-off before heel-strike and slowing down. c) Analogous to panel-c, except the walking speeds up during the transition. d) Velocity changes and impulses when the heel-strike precedes push-off entirely.

OC $\tan\theta_{\text{after}}$,

$$\text{ED} = \frac{\text{OA}}{\cos\theta_{\text{before}}} - \frac{\text{OC}}{\cos\theta_{\text{after}}}, \quad \frac{\text{BD}}{\text{ED}} = \frac{\cos\theta_{\text{after}}}{\sin(\theta_{\text{after}} + \theta_{\text{before}})}, \quad \text{and} \quad \frac{\text{EB}}{\text{ED}} = \frac{\cos\theta_{\text{before}}}{\sin(\theta_{\text{after}} + \theta_{\text{before}})} \tag{2}$$

in Eq. 1 to obtain:

$$W_{\text{pos}} = \frac{1}{2}m\left[V_{\text{before}}\tan\theta_{\text{before}} - \frac{\cos\theta_{\text{after}}}{\sin(\theta_{\text{after}} + \theta_{\text{before}})}\left(\frac{V_{\text{before}}}{\cos\theta_{\text{before}}} - \frac{V_{\text{after}}}{\cos\theta_{\text{after}}}\right)\right]^2 \text{ and} \tag{3}$$

$$W_{\text{neg}} = \frac{1}{2}m\left[V_{\text{after}}\tan\theta_{\text{after}} + \frac{\cos\theta_{\text{before}}}{\sin(\theta_{\text{after}} + \theta_{\text{before}})}\left(\frac{V_{\text{before}}}{\cos\theta_{\text{before}}} - \frac{V_{\text{after}}}{\cos\theta_{\text{after}}}\right)\right]^2, \tag{4}$$

with OA $= V_{\text{before}}$ and OC $= V_{\text{after}}$. We obtain exactly the same expressions for speeding up during step-to-step transition as shown in figure S1c (though the figure looks superficially different, a couple of negative signs cancel, thereby giving the same answers). Note that the main qualitative difference between panels b and c of figure S1 is that the velocity $\overrightarrow{\text{OB}}$ is above or below the horizontal.

An implicit assumption in the above derivation is that the push-off and heel-strike impulses do not require tensional leg forces (the leg cannot pull on the ground) and are in the directions shown. This requirement is satisfied when the ratio of the two speeds obey the following condition:

$$\cos(\theta_{\text{after}} + \theta_{\text{before}}) \leq \frac{V_{\text{after}}}{V_{\text{before}}} \leq \frac{1}{\cos(\theta_{\text{after}} + \theta_{\text{before}})}.$$

When $V_{\text{after}}/V_{\text{before}} = \cos(\theta_{\text{after}} + \theta_{\text{before}})$, the necessary push-off impulse becomes zero and when $V_{\text{before}}/V_{\text{after}} = \cos(\theta_{\text{after}} + \theta_{\text{before}})$, the necessary heel-strike impulse becomes zero.

Heel-strike before push-off. When the heel-strike impulse precedes the push-off impulse, the negative work W_{neg} by the heel-strike and the positive work by the push-off W_{pos} are given by the respective kinetic energy changes:

$$W_{\text{neg}} = \frac{1}{2}m\left(\text{OA}^2 - \text{OG}^2\right) \quad \text{and} \quad W_{\text{pos}} = \frac{1}{2}m\left(\text{OC}^2 - \text{OG}^2\right), \tag{5}$$

where OA $= V_{\text{before}}$, OC $= V_{\text{after}}$, OG$^2 = $ OQ$^2 + $ QG^2, OQ $= $ OA $\cos(\theta_{\text{after}} + \theta_{\text{before}})$, angle $\angle$QGC $= \theta_{\text{after}} + \theta_{\text{before}}$, QG $= $ CG $\cos(\theta_{\text{after}} + \theta_{\text{before}})$, and CG $= $ AB in figure S1b. i.e.,

$$\text{CG} = \text{AB} = V_{\text{before}}\tan\theta_{\text{before}} - \frac{\cos\theta_{\text{after}}}{\sin(\theta_{\text{after}} + \theta_{\text{before}})}\left(\frac{V_{\text{before}}}{\cos\theta_{\text{before}}} - \frac{V_{\text{after}}}{\cos\theta_{\text{after}}}\right)$$

using the derivation for push-off before heel-strike; the final expression is easily obtained by substituting these relations into equation 5. As has been shown before for steady state walking [4, 2], we find that a transition with heel-strike before push-off requires more metabolic cost than push-off before heel-strike even when changing speeds with the simplifying assumption that $\theta_{\text{before}} = \theta_{\text{after}}$.

S2 Optimal multi-step inverted pendulum gaits satisfying experimental protocol

We use a metabolic cost that is a sum of two terms: (1) the step-to-step transition cost and (2) a swing cost.

Step-to-step transition cost. The step-to-step transition cost E_{s2s} is a weighted sum of the push-off work W_{pos} and heel-strike work W_{neg}, summed over all steps, and scaled by the approximate efficiencies of positive and negative work respectively:

$$E_{\text{s2s}} = \sum_{\text{steps}} \eta_{\text{pos}}^{-1}W_{\text{pos}} + \eta_{\text{neg}}^{-1}W_{\text{neg}}$$

3

using the equations 3 and 4 for the work expressions.

Swing cost. The step-to-step transition cost does not account for the work required to swing the legs. We use a simple model of the metabolic cost required to swing the legs forward [5] equal to

$$E_{\text{swing}} = \mu D_{\text{stride}}/T_{\text{step}}^3$$

where T_{step} is the step duration, D_{stride} is the distance travelled by the swing foot during the step (distance between previous and successive foot contact points), and the proportionality constant $\mu = 0.06$ when all other quantities are non-dimensional, chosen so as to best fit steady walking metabolic costs [5].

Representing a multi-step inverted pendulum walking motion. Each step of the inverted pendulum walking motion was represented using five variables: the initial leg angle θ_0, the initial (post-heel-strike) angular velocity $\dot{\theta}_0$, step duration T_{step}, the constant leg-length over the step ℓ_{leg}, and the foot-ground contact position in the forward direction x_{contact}. Nonlinear equality constraints make sure that the body position at the end of one step is equal to that at the beginning of the next step.

Numerical optimization. We used numerical optimization to determine the multi-step walking motion that satisfies the oscillating-speed experimental protocol and minimizes the model metabolic cost as described above. The biped model alternates between a higher speed $v_{\text{avg}} + L/T_{\text{fwd}}$ and a lower speed $v_{\text{avg}} - L/T_{\text{bck}}$, each lasting a few steps, so that the net average speed is v_{avg}, and the forward and backward movement in lab frame have periods equal to T_{fwd} and T_{bck}. The number of steps for the forward and backward movements are chosen based on the number of steady walking steps in the durations T_{fwd} and T_{bck}. Other constraints included an upper bound on the leg length ($< \ell_{\text{max}}$) and a periodicity constraint on the body height over one period of back and forth walking. The optimization problem was solved in `MATLAB` using the optimization software `SNOPT`, which employs the sequential quadratic programming technique [6]. At each average speed, we also computed the optimal constant-speed inverted pendulum walking gait (repeating calculations in [3, 7]), so as to subtract from the optimal oscillating-speed walking cost.

Leg force cost. We repeated the calculations above with a cost for leg force, proportional to the integral of the leg force, with a proportionality constant as in [8]. We found that this leg force cost did not change our overall predictions for the difference between oscillating-speed and constant-speed metabolic costs, as both these costs increase by the almost same amount due to the leg force cost. This result can be explained intuitively as follows: because the legs make relatively small angle with the vertical, as explained in [7], the average leg force is approximately equal to the average vertical force, which has to be equal to the total body weight for periodic motion – be it constant-speed walking or oscillating-speed walking.

S3 Daily energy budget for starting and stopping

Subjects in [9] performed a majority of the walking over a day in short bouts; they walked in 43914 bouts and took a total of 1717730 steps. Assuming a typical step length of 0.6 m [10], the subjects walked a total distance of 1030638 m. Assuming the subjects walked the whole distance at a constant speed of 1.4 m/s, we can predict a total constant-speed energy expenditure to be 2262600 J, based on a parabolic relationship given by $\dot{E}_{\text{steady}} = a + bv^2$ with $a = 2.22$ W/kg and $b = 1.15$ W/kg/(ms^{-1})^2 [11]. But, such a cost would ignore the cost of accelerating from and to rest at the start and the end of the bout. We can approximate this daily unsteady cost for the 43914 bouts to be 137380 J by extrapolating our results from the kinetic energy-based model with the unsteady cost for one bout given by, $\lambda_{ke}(\eta_{\text{pos}}^{-1} + \eta_{\text{neg}}^{-1})v^2/2$, where $\lambda_{ke} = 0.67$. The ratio of the cost of changing speed to cost of walking is thus found to be 0.06, in other words, the unsteady cost of walking per day is 6% of the steady cost on average. Using the 95% C.I. for λ_{ke} gives us 4-8% as reported in the main manuscript. This approximate calculation shows that the cost of changing speeds is a significant fraction of the energy humans consume in daily walking.

S4   Comparison with a previous study

Our oscillating speed protocols had speed fluctuations between ± 0.13 and ± 0.27 m/s. As noted in the main manuscript, one previous article [12] attempted to measure the cost of changing speeds, with greater speed fluctuations (± 0.15 to ± 0.56 m/s) and higher kinetic energy fluctuations per unit time than our study. The rate of kinetic energy fluctuations for both experiments can be compared by comparing $v\Delta v/T$, for a fluctuation between speeds $v - \Delta v$ and $v + \Delta v$ in T seconds. In [12] $v\Delta v/T$ ranges between to 0.0226 and 0.127 m^{2}s^{-3} and in our protocol, $v\Delta v/T$ ranges between 0.026 and 0.1108 m^{2}s^{-3}. Thus, the kinetic energy fluctuation rates were similar in the two studies. Nevertheless, the study [12] found significant increase only for their highest speed fluctuation but not lower. As noted in the main manuscript, this study [12] required walking on oscillating-speed treadmill belts or controlling step durations in overground walking (derived from oscillating-speed treadmill). An oscillating-speed treadmill, being a non-inertial reference frame, can perform mechanical work on the subject, and is not mechanically equivalent to oscillating-speed walking overground. Further, controlling step durations [12] will produce incorrect speed fluctuations that do not obey the speed-step-duration relation for directly controlling walking speed, as established by Bertram and Ruina [13] in the case of steady walking.

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